SLOWING DOWN

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Abstract

We extend the efficiency wage model of Shapiro and Stiglitz to account for the observation that workers’ effort has a tendency to fall when they approach the end of their employment contract. In particular, we find that the efficiency wage increases when the end of term approaches for a given rate of unemployment. We draw implications for the behavior of workers who are approaching retirement, temporary employment contracts, and the advance notice of impending job loss.

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1. Introduction

Every employment contract has a time dimension. Workers on temporary contracts see their contracts expire; workers who have been given an advance notice of dismissal know that their days on the job are numbered; and even workers who have safe permanent jobs realize that they will eventually retire. In this paper we show how the expected end of tenure may affect workers’ effort. In particular, we extend the well-known model of wage setting by Shapiro and Stiglitz (1984) to show how a worker’s propensity to shirk his duties varies from the beginning to the end of an employment contract.

The time path for the wages required to deter shirking within the Shapiro and Stiglitz model can help explain various real life phenomena. Young university lecturers frequently complain about older colleagues who are not engaged in research. Workers may be forced to retire at a certain age or once their productivity has started to fall and these individuals may then move to professions where opportunities for on-the-job leisure are greater. Politicians sometimes end up as diplomats; football players as celebrities and movie stars may take on fewer roles and end up enjoying leisure and fame. In some cases the decision is driven by physical deterioration, such as in sports, but in other cases it is for other less well-defined reasons such as when an academic stops spending his time doing research.¹

¹ Even in the case of athletes, the retirement decision is to some extent up to the individual’s discretion because the rate of deterioration of physical ability tends to be quite small. This has been demonstrated in many studies, such as Fair (1994, 2007) who fails to find a strong effect of aging on physical abilities.

We start by reviewing some of the evidence for the relationship between contract length and effort before moving to the theoretical extensions. A final section discusses some of the implications of the model.

2. From sports to academics

The field of sports offers an opportunity to study how performance of individuals varies over time. In a recent paper, Krautmann and Solow (2009) examine incentives in baseball contracts and find that players who are less likely to sign a subsequent contract show worse performance while those anticipating signing another contract perform much better.
Cain (2011) studies hockey players and finds that a player’s performance decreases as his likelihood of retirement at the end of the current contract increases.

Another field with observable performance is politics. The voting behavior of politicians has been studied extensively by political scientists. One form of shirking in politics is to change one’s voting pattern so that it no longer conforms to the preferences of voters. Figlio (1995) finds that the decision to retire from politics results in changed political behavior or political shirking using a multi-year panel data set. Tien (2001) finds that shirking exists among voluntarily retiring members of Congress. Parker and Powers (2002) find that members of Congress who are about to leave office have a tendency to spend more on foreign travel. DeBacker (2012) detects shirking by senators in their last term that is limited by political parties that constrain the politician in his last term to a varying extent depending on their post-Senate career choices. This finding is supportive of Lott (1990) who finds that shirking can be reduced when opportunities exist for political parties to affect a shirking politician’s post-elective career.

There are studies of the relationship between age and productivity in firms. Dostie (2006) uses Canadian data and finds that productivity profiles is concave in age. Börsch-Supan et al. (2007) study productivity in a German car manufacturing company and find a negative effect of age on productivity. Lallemand and Ryckx (2009) use productivity data from large Belgian firms and show that a higher share of older workers lowers average productivity. Van Ours (2009) uses matched worker-firm data from the Netherlands and finds that when the average age of the workforce within a firm goes up productivity goes down. Göbel and Zwick (2012) analyze the impact of changes in the age structure of establishments on productivity using employer-employee panel data for the Netherlands. They find that there are no significant differences in the age-productivity profiles between the manufacturing and the services sectors as could be expected since the former may require greater physical strength.

At the university level, there is statistical evidence showing that research productivity is declining in age. Oster and Hamermesh (1998) find that economists’ productivity measured by publications in leading journals declines with age, although the probability of acceptance, once an article has been submitted to a leading journal, is independent of age. Moreover they find that the median age of authors of articles in leading economics
journals was 36 in the 1980s and the 1990s and that a very small minority of authors are over 50 in spite of a substantial percentage of AEA members being over the age of 50.² However, they cannot discriminate between the two possible reasons for this observation; whether the falling frequency of publications is due to deteriorating mental faculties or, alternatively, reflects rational decisions to devote less time to research. In a recent paper, Jones (2010) analyses the age of individuals at the time of their greatest achievements in science using data on research that leads to the Nobel Prize in physics, chemistry, medicine and economics and also data on research that leads to great technological achievements as shown in the almanacs of the history of technology. He finds that the greatest concentration of innovations in the life of a scientist comes in the 30s but a substantial amount also comes in the 40s, while scientists in their 50s, and even more so in the 60, generate far fewer discoveries.

We now move to the extension of the Shapiro-Stiglitz model – S-S from now on – and then discuss the implications of the model for wages and unemployment.

3. Shirking with finite horizons

We model a worker’s effort decision when he has finite horizons. In other words he realizes that the end of his contract or, alternatively, working life is gradually approaching. In the Shapiro-Stiglitz (1984) model this realization affects his decisions to shirk. The extension of the model will leave the infinite horizon case described in the S-S paper as a special case.

There are three states of intertemporal utilities in the S-S model for workers with transitory probabilities to alternative states. These are the value of being employed, \( V_E \) (when not shirking) and \( V_S \) (when shirking), and the value of being unemployed, \( V_U \). Workers receive the wage \( w \) when employed and unemployment benefits \( b_u \) when unemployment. Effort \( \bar{e} \) is exerted when employed workers are not shirking their duties while no effort is exerted when workers shirk. Workers discount future utility at rate \( \rho \),

² Similar results are reached by Lehman (1953), Diamond (1986), McDowell (1982) and Levin and Stephan (1992) for other disciplines. However, Jan van Ours (2009) finds no relationship between the quality-adjusted rate of publication and age among his colleagues at Tilburg University.
face a constant probability of job termination $b$ during the contract period and the probability $q$ of being fired if shirking.

We start with a representative state $i$

$$V_i = \int_i^- u_i(s)e^{-\rho(s-t)}ds,$$  

(1)

with transitory probability $p_{ij}$ of moving to the alternative state $V_j$, where $u_i(s)$ is the immediate utility at time $s$ for the state $i$. We can now introduce finite horizons by dividing the inter-temporal integral $V_i$ into the periods of $t \leq time \leq T$ and $T \leq time \leq \infty$:

$$V_i = \int_t^- u_i(s)e^{-\rho(s-t)}ds = \int_T^- u_i(s)e^{-\rho(s-t)}ds + \int_T^\infty u_i(s)e^{-\rho(s-t)}ds.$$  

(2)

The integral $\int_T^\infty u_i(s)e^{-\rho(s-t)}ds$ for time period $T \leq time \leq \infty$ can be rewritten as follows

$$\int_T^\infty u_i(s)e^{-\rho(s-t)}ds = e^{-\rho(T-t)}\int_t^- u_i(s)e^{-\rho(s-t)}ds.$$  

(3)

Therefore, we need to discount the integral by the factor $e^{-p_{ij}(T-t)}$ if we would like to replace $T$ with $t$ since over the time period from $t$ to $T$, the integral $\int_T^\infty u_i(s)e^{-\rho(s-t)}ds$ depreciates at the rate of $p_{ij}$:

$$e^{-\rho(T-t)}\int_t^- u_i(s)e^{-\rho(s-t)}ds = e^{-(\rho(p_{ij})(T-t))}\int_t^- u_i(s)e^{-\rho(s-t)}ds.$$  

(4)

Equation (2) can now be rewritten as

$$V_i = \int_t^- u_i(s)e^{-\rho(s-t)}ds + e^{-(\rho(p_{ij})(T-t))}\int_t^- u_i(s)e^{-\rho(s-t)}ds = V_i^T + e^{-(\rho(p_{ij})(T-t))}V_i,$$  

(5)

where $V_i^T = \int_t^T u_i(s)e^{-\rho(s-t)}ds$. Rearranging gives

$$V_i = \frac{V_i^T}{1 - e^{-(\rho(p_{ij})(T-t))}}.$$  

(6)

Equation (6) shows the relationship between the perpetual and non-perpetual intertemporal integrals for the state $i$. One can then apply equation (6) to three states: $V_E$, $V_S$, and $V_U$, with corresponding transitory probabilities: $p_{EU} = b$, $p_{SU} = b+q$, and $p_{UE} = a$. 

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where \( V_E^T = \int_t^T (w - \bar{\tau}) e^{-\rho(s-t)} \, ds \) is the non-perpetual integral for the value of being a non-shirking employed worker who faces the probability \( b \) of moving to the unemployed state,

\( V_S^T = \int_t^T w e^{-\rho(s-t)} \, ds \) is the non-perpetual integral for the value of being a shirking worker who faces the probability \( b+q \) of moving to the unemployment state and

\( V_U^T = \int_t^T b_s e^{-\rho(s-t)} \, ds \) is the non-perpetual integral for an unemployed worker who becomes employed with probability \( a \), which denotes the probability of finding jobs.

We can derive the following three asset pricing equations by substituting equation (6) into the Bellman equations of the perpetual case of the S-S model;

\[
\rho V_E^T = \left( w - \bar{\tau} \right) \left( 1 - e^{-\rho b |T-t|} \right) + b \left( V_U^T \frac{1}{1-e^{-\rho a |T-t|}} - V_E^T \right),
\]

(7)

\[
\rho V_S^T = w \left( 1 - e^{-\rho b+q |T-t|} \right) + (b+q) \left( V_U^T \frac{1}{1-e^{-\rho a |T-t|}} - V_S^T \right),
\]

(8)

\[
\rho V_U^T = b_s \left( 1 - e^{-\rho a |T-t|} \right) + a \left( V_E^T \frac{1}{1-e^{-\rho b |T-t|}} - V_U^T \right).
\]

(9)

Using the no-shirking condition such that \( V_E^T = V_S^T \) for equation (8) gives

\[
\rho V_E^T = w \left( 1 - e^{-\rho b+q |T-t|} \right) + (b+q) \left( V_U^T \frac{1}{1-e^{-\rho a |T-t|}} - V_E^T \right).
\]

(10)

There are three unknown variables, \( V_E^T, V_U^T, w \), for (7), (9) and (10). Rearranging those three equations gives

\[
(\rho + b) V_E^T - b \left( V_U^T \frac{1}{1-e^{-\rho a (T-t)}} \right) - \bar{\tau} \left( 1 - e^{-\rho b |T-t|} \right) = 0,
\]

(11)

\[
(\rho + b + q) V_E^T - (b+q) \left( V_U^T \frac{1}{1-e^{-\rho a |T-t|}} \right) - w \left( 1 - e^{-\rho (b+q) |T-t|} \right) = 0,
\]

(12)
Finally, using Cramer’s rule gives the no-shirking condition for wages (see Appendix for details)

\[
w = \frac{\left(1 - B/A\right)\left[\bar{e}a(b + q) - b_u(b(\rho + b + q))\right] + \left(B/A\right)b_u\rho q + \bar{e}\rho (a + b + \rho + q)}{\left(1 - B/A\right)\left[(\rho + b)(\rho + a) + a\bar{q}\right] + \rho q},
\]  

(14)

where \( A = \left(1 - e^{-(a+b)(T-t)}\right) \) and \( B = \left(1 - e^{-(\rho+b+q)(T-t)}\right) \). Note that since \( A < B \) we find that \((1 - B/A)\) is negative. The numerator of (14) falls faster than the denominator and the firm needs to pay wages that rise as the end of the contract period approaches. Because the effective discount rates for the shirking state is \( \rho + b + q \) and higher than the effective discount rate for the non-shirking state \( \rho + b \), shirking is less harmful to workers whose contract will expire soon.

For the perpetual case, we have \( A = B \). Thus the no-shirking condition becomes

\[
w = \frac{\rho \bar{e} + \rho (\rho + a, \rho + \bar{q})}{\rho \bar{q}} = \frac{\rho \bar{e} + \rho (\rho + a, \rho + \bar{q})}{\rho \bar{q}},
\]

(15)

which is the original no-shirking condition of Shapiro and Stiglitz.

4. Retirement

The derivation above can be used to model age-dependent wage setting. Lazear (1979) proposed the idea that older workers who were promised rising wage profiles at the beginning of their job tenure end up with wages exceeding their productivity which then calls for their mandatory retirement.\(^3\) In our extension of the S-S model we find that expected remaining tenure is finite and older workers will either have to be paid more or face higher unemployment rates in order to induce them not to shirk. The higher wages are necessary not because rising wage profiles are used to create incentives throughout

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\(^3\) Sala-i-Martin (1995) offered a related economic rationale for mandatory retirement. In his model, mandatory retirement serves the purpose of preventing older, low-productivity workers from lowering the productivity of their younger colleagues in the workplace.
one’s career as in Lazear (1981) but because only at a higher wage do they refrain from shirking realising how short their remaining tenure is.

Denote the number of employed workers of age $t$ by $L_t$. In steady state, the outflow from employment to unemployment equals $bL_t$ and should equal to inflow of workers from unemployment to employment $a(N_t - L_t)$ where $N_t$ is the number of workers of age $t$ in the labor force.

$$bL_t = a(N_t - L_t).$$

(16)

Thus $a + b = bL_t (N_t - L_t)^{-1} + b = bN_t (N_t - L_t)^{-1} = bN_t (N_t - L_t)^{-1} = b/u_t$ and we find $a = b(1-u_t)/u_t$. Substituting back into (14) gives the no-shirking condition in equilibrium as a relationship between wages and unemployment.

$$w = \frac{(1-B/A)[b((1-u_t)/u_t)(b+q) - b\rho b(\rho + b + q)]}{(1-B/A)[(\rho + b)((1-u_t)/u_t) + b((1-u_t)/u_t)q + \rho q] + (B/A)\rho(1-b(1-u_t)/u_t) + \rho q}.$$

(17)

It follows that each cohort of workers has a distinct wage curve – or no-shirking constraint – described by equation (17).

The non-shirking constraint is drawn in Figure 1 below as an upward-sloping wage curve or a non-shirking constraint for different age groups. The benchmark values are given below the figure.

**Figure 1.** Age-dependent wage curves
There are only small differences between young and middle-aged workers. But the wage curves for older workers are substantially higher. It follows that the wage – or unemployment – needed to prevent a 40-year old worker from shirking his duties is not much higher than that needed to prevent a 20 year old worker from doing so but a significantly higher wage is needed to prevent a 50 year old worker from shirking than is the case of the 40 year old one.

As shown in Figure 2 we find that when all age groups belong to the same labor market and suffer identical unemployment rates that the wage required to prevent shirking rises rapidly in the 44-48 years age group when unemployment is 10%, in the 48-52 years age group when unemployment is 20% and in the 52-56 group when unemployment is a staggering 40%. Our results are consistent with the observation that employment-to-population ratios are significantly lower for the 60-64 years age group than for the 15-64 years group in all OECD countries.  

Figure 2. The non-shirking wage and age

Parameter values: \(\rho = 0.1, b = 0.1, q = 0.3, e = 1.0, b_e = 1, N = 1000\). Note that \(T=45\) implies that age = 20; and \(T=5\) means that age = 60, if we assume that age 65 is the age at which workers are no longer willing to work.

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4 The rates are 68.26% in France for the 15-64 year olds in 2010 and 19.13 for the 60-64 year olds. Similar numbers are 76.11 (49.26) for Germany, 67.68 (29.56) for Italy, 80.01 (48.15) for the Netherlands, 75.27 (54.19) for the UK and 71.05 (55.12) for the US, See OECD statistics portal, www.oecd.org.
5. Conclusions

We have shown how, for a given wage and rate of unemployment, workers put less value on their jobs the shorter their remaining tenure is. This increases their incentive to shirk their duties, which then requires firms to raise wages at a given level of unemployment. The same intuition applies to the analysis of temporary contracts and young workers with short expected job tenures. The shorter the contract, the lower is the value of the job to the worker and the more likely is he to shirk his duties. Similarly, a worker who has been given an advance notice of impending job loss is likely to lower his effort, which could explain why advance notice is not more common.5

5 See Kuhn (1992) and Addison and Chilton (1997).
References


Appendix

Equations (11)-(13) can be written as follows:

(A1)  \((\rho + b)V^T_e - b\frac{A}{C}V^T_u - Aw = -A\bar{e}\),

(A2)  \((\rho + b + q)V^T_e - (b + q)\frac{B}{C}V^T_u - Bw = 0\),

(A3)  \(a\frac{C}{A}V^T_e - (\rho + a)V^T_u = -Cb\),

where \(A = \(1 - e^{-(\rho + b)(T-t)}\)\), \(B = \(1 - e^{-(\rho + b + q)(T-t)}\)\), and \(C = \(1 - e^{-(\rho + a)(T-t)}\)\).

Cramer’s rule gives the solutions for no-shirking conditions wages

\[
w = \frac{\begin{vmatrix}
(\rho + b) & -b\frac{A}{C} & -A\bar{e} \\
(\rho + b + q) & -(b + q)\frac{B}{C} & 0 \\
a\frac{C}{A} & -(\rho + a) & -Cb
\end{vmatrix}}{\begin{vmatrix}
(\rho + b) & -b\frac{A}{C} & -A \\
(\rho + b + q) & -(b + q)\frac{B}{C} & -B \\
a\frac{C}{A} & -(\rho + a) & 0
\end{vmatrix}}.
\]

Expanding the determinants gives

(A5) \(w = \frac{Bb_n(\rho + b)(b + q) + A\bar{e}(\rho + b + q)(\rho + a) - Ba\bar{e}(b + q) - Abb_n(\rho + b + q)}{A(\rho + b + q)(\rho + a) + Bab - Ba(b + q) - B(\rho + b)(\rho + a)}\).

Equation (A5) can further be simplified as follows,

\[
w = \frac{(A-B)b_n(\rho + b)(b + q) + Bb_n\rho q + (A-B)\bar{e}(b + q) + A\bar{e}\rho(\rho + b + q)}{(A-B)[(\rho + b)(\rho + a) + aq] + Apq}
\]

(A6) \(= \frac{(A-B)[\bar{e}(b + q) - b_n(\rho + b + q)] + Bb_n\rho q + A\bar{e}\rho(\rho + b + q)}{(A-B)[(\rho + b)(\rho + a) + aq] + Apq}
\]

\(= \frac{(1 - B/A)[\bar{e}(b + q) - b_n(\rho + b + q)] + (B/A)b_n\rho q + \bar{e}\rho(\rho + b + q)}{(1 - B/A)[(\rho + b)(\rho + a) + aq] + \rho q}
\).