Threshold Effects of Dismissal Protection Regulations and Employment Dynamics

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Abstract

Labour market regulations aimed at enhancing job-security are dominant in several OECD countries. These regulations seek to reduce dismissals of workers and fluctuations in employment. The main theoretical contribution is to gauge the effects of such regulations on labour demand across establishment sizes. In order to achieve this, we investigate an optimising model of labour demand under uncertainty through the application of real option theory. The calibration results indicate that labour market rigidities may be crucial for understanding sluggishness in firms’ labour demand across plant sizes in continental Europe.

Keywords: Labour Demand, Dismissal Protection Legislation, Firing Costs, Real Options
JEL-Classification: J23, J58, D81
1. Introduction

In many continental European countries unemployment appears to reside at a persistently high level, with no improvement in sight. Therefore, protection of workers from dismissals has become an important topic of labour markets reforms in many European countries. According to the World Bank Doing Business database, countries vary greatly with respect to the flexibility of labour market regulations. These regulations can be provided through legislation, collective bargaining agreements or judicial practices and court interpretations of legislative provisions. According to the World Bank, for example, severance pay in Germany is set at 66.7 weekly wages, in the Netherlands at 16.0 weekly wages, in the UK at 33.5 weekly wages, and Portugal requires 98.0 weekly salaries as the standard compensation. On the contrary, the corresponding number for the U.S. is 0.0. Given these differences the pros and cons of deregulating labour markets are at the heart of the employment debate in many countries.

An important characteristic of dismissal protection laws or collective agreements in advanced economies is that rules for dismissing redundant workers are differentiated by establishment size and the provisions are more stringent above certain employee thresholds. In Germany, the threshold in the “Protection Against Dismissal Act” (Kündigungsschutzgesetz) was changed several times. During the 1990s, the threshold was changed twice, once from 5 to 10 (full-time equivalent) employees in October 1996 by the then chancellor Helmut Kohl and then back again to 5 employees in January 1999 under chancellor Schröder. Finally, in January 2004 the threshold was moved once again from 5 to 10 employees. The size exemption criteria apply to establishments, not firms. An establishment is a production unit at a single location which can financially and/or legally belong to a larger firm. Establishments below the threshold are allowed to operate under the far less stringent rules of the German Civil Code (Bürgerliches Gesetzbuch). The corresponding Austrian threshold level for

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1 The World Bank Doing Business scoreboard on the flexibility of labour regulations and their enforcement is available at [www.doingbusiness.org/ExploreTopics/HiringFiringWorkers/](http://www.doingbusiness.org/ExploreTopics/HiringFiringWorkers/). The table provides five indicators for a worker in a large manufacturing firm who has been with the company for many years. (1) Difficulty of hiring a new worker (Difficulty of Hiring Index); (2) restrictions on expanding or contracting the number of working hours (Rigidity of Hours Index); (3) difficulty and expense of dismissing a redundant worker (Difficulty of Firing); (4) an average of the three indices (Rigidity of Employment Index), and (5) cost of a redundant worker, expressed in weeks of wages (Firing Costs). Higher values in the table indicate more rigid regulations. Also see Botero et al. (2004). The OECD has also published indices of employment protection, again showing less protection in English-speaking countries [see OECD (2004)].

2 A recent proposal for reform of the German “Protection Against Dismissal Act” has suggested that parties should be allowed to agree in advance that in case of dismissals on economic grounds the employee waives protection and claims a statutory redundancy payment instead.

3 In principle, this provides an opportunity for some firms to engage in strategic behaviour. If exemption is possible on an establishment basis, then larger firms can be split up into several establishments to elude the legal constraints [see Borgarello et al. (2003)].
application of dismissal protection is 5 workers, while no such thresholds currently exist in the UK and the Netherlands. The potential importance of the 5-employee-threshold vs. 10-employee-threshold in the “Protection Against Dismissal Act” results from the fact that Germany’s economy is dominated by small and medium-sized firms, the Mittelstand, which has often been described as the backbone of the German economy.

Thus far, the empirical evidence concerning the impact of these thresholds on firms is rather scant. Bauer et al. (2004) have investigated the impact of Germany's dismissal protection legislation on employment in small establishments using a matched employer-employee data set. They find that the stringency of this legislation has no significant effect on labour turnover in such establishments. As for other countries, Boeri and Jimeno-Serrano (2005) study the impact of the 15-employee-threshold in the Italian dismissal protection legislation and also do not find any impact on employment. Verick (2004) presents some evidence that the loosening of the dismissal protection exemption threshold in Germany was associated with less employment. Finally, Messina and Vallanti (2005) show that more stringent firing restrictions dampen the response of job destruction and job creation to business cycles in 14 European countries.

Our aim is to model the effects of such thresholds upon labour demand. To this end, we construct a real options model of labour demand with threshold effects. This is still a blank cell in the real options modelling literature.

The remainder of the paper is structured as follows. Section 2 develops the baseline theoretical model of employment dynamics and demonstrates the implications of various policy reforms. Section 3 extends the model to allow for endogenous wages and productivity with illustrative numerical examples. Section 5 concludes. Two appendices at the end of the paper collect some proofs and technical derivations which are rather involved. Readers who are not interested in the nuts and bolts of the derivations, can skip the appendices without losing the main argument of the paper.

2. The Baseline Modelling Framework

In the case of completely reversible employment decisions, a hiring (firing) decision is made if the wage is larger (smaller) that the marginal product of labour. As is widely acknowledged in the literature, this conventional valuation technique is no appropriate tool for factor demand decision making in the presence of uncertainty and (partial) irreversibility. The reason is that the traditional rule considers hiring and firing decisions only as being of a “now or never” nature.

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4 French and Spanish laws do not exclude small businesses from dismissal protections but reduce their obligations with respect to severance pay (France: no compulsory compensation; Spain: reimbursement of severance pay from a fund).
With the appearance of the real options theory, the implication of uncertainty and partial cost irreversibility on factor demand decisions are well emphasised. In this section we therefore construct a real options model for employment under uncertainty. The stochastic framework contains the threshold effect induced by the “Protection Against Dismissal Act” to advance our understand of the impact of the institutional setting. Like other real option models of this type, the optimal employment policy is a trigger strategy such that hirings and firings are initiated when the marginal product of labour reaches a critical threshold.\(^6\) We believe this to be an appropriate framework for understanding the impact of threshold levels for application of dismissal protection on employment, while still yielding tractable results.\(^7\)

We first characterise the optimal employment strategy of an imperfect competitive firm subject to idiosyncratic shocks and firing and hiring costs, holding wages and productivity constant. The starting point is the Cobb-Douglas production function

\[
Y_t = AK^\alpha L_t^{1-\alpha}, \quad 0 < \alpha < 1,
\]

where \(K\) is the capital stock, \(L_t\) is the employment level, \(\alpha\) is a parameter determining the shares between capital and labour in production, and \(A\) represents the level of technology. It is assumed that the firm faces an isoelastic demand function subject to multiplicative demand shocks

\[
p = Y_t^{(1-\psi)/\psi} Z_t, \quad \psi \geq 1,
\]

where \(p\) denotes the price, \(Y_t\) is real output, \(Z_t\) denotes the multiplicative stochastic demand shock, and \(\psi\) is an elasticity parameter that takes its minimum value of 1 under perfect competition. Therefore, the profits at \(t\), \(\Pi_t\), measured in units of output, are defined as

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\(^5\) In 2001 about two-third of all German establishments had 1 to 5 employees. See EUROSTAT (2001) for further details. According to the German Statistical Office even 91 percent of all German firms had 1-9 employees in 2006 (see [http://www.destatis.de/basis/d/insol/unternehmentab2.php](http://www.destatis.de/basis/d/insol/unternehmentab2.php)).

\(^6\) In its methodological approach, the model comes within the scope of the real options literature which has developed rapidly over the past decade. Reviews of this burgeoning literature are provided in Copeland and Antikarov (2001), Copeland and Tufano (2004), Coy (1999) and Dixit and Pindyck (1994).

\(^7\) The model does not pretend to be a complete picture of the economy but rather to capture important key features that matter for labour market policies. For example, we have ignored behavioural assumptions regarding market rivalry, which in turn would necessitate some kind of game-theoretic analysis to take account of the strategic interactions among the firms, results of which are in turn heavily dependent on assumptions regarding the information sets available and the type of game being played. Leahy (1993) has shown that the assumption of myopic firms who ignore the impact of other firms’ actions results in the same critical boundaries that trigger investment as a model in which firms correctly anticipate the strategies of other firms. Grenadier (2002) has recently extended Leahy’s (1993) “Principle of Optimality of Myopic Behavior” to the apparently more complex case of dynamic oligopoly under uncertainty. Both papers therefore permit one to bypass strategic general equilibrium considerations when analysing factor demand under uncertainty.
\( \Pi_i = A^{\psi} Z_i K^{-\alpha_1} L_{t_1}^{\alpha_2} - wL_i - C(M_i, L_i), \)

where \( \alpha_1 = \frac{\alpha}{\psi} \) and \( \alpha_2 = \frac{(1 - \alpha)}{\psi} \), \( wL_i \) denotes the total wage bills paid by the firm, \( M_i \) represents gross employment changes due to hiring or firing and quits from employees, and \( C(\cdot) \) are the total employment adjustment expenditures. Following Nilsen et al. (2003), there are asymmetric fixed, proportional, and convex costs of adjusting employment in either direction.\(^8\) More specifically:

\[
C(M_i) = \begin{cases} 
  c_h + p_h M_i + \frac{1}{2} \gamma_h M_i^2 & \text{for } M_i > 0, \\
  0 & \text{for } M_i = 0, \\
  \frac{c_f}{1 + e^{-\gamma_0(L_i - l)}} - p_f M_i + \frac{1}{2} \gamma_f M_i^2 & \text{for } M_i < 0.
\end{cases}
\]

There is an economic meaning behind these three cost components. (1) Hiring and firing employees incur some proportional positive unit costs of hiring and firing, \( p_h \) and \( p_f \), respectively.\(^9\) Firing employees does generate some positive costs per employee: \(- p_f M_i > 0 \) for \( M_i < 0 \). The positive unit costs of hiring and firing also reflect the (partial) irreversibility of employment changes; (2) the convex cost functions reflect the adjustment and disruptions to production processes; in case of asymmetric convex costs marginal cost of hiring are not the same as the marginal costs of firing; (3) the fixed costs of hiring and firing are related to advertising and screening and are set up to a point independent of the number of people hired. The costs also include fixed costs of legal consultation and disputes in case of firings. In addition to explicit costs, a change in the level of employment is likely to involve implicit costs in terms of temporary productivity losses; (4) moreover, there are no costs as long as no hirings/firings are made, or equivalently, \( C(0) = 0 \).

The novelty of our model is that we allow for heterogeneity among firms by formalizing the threshold levels for application of dismissal protection which exist in many countries. We assume that the fixed cost of firing can be depicted by a three-parameter logistic function, \( c_f \left/ (1 + e^{-\gamma_0(L_i - l)}) \right. \), where \( l \) denotes the threshold level for application of dismissal protection, \( c_0 \) is a scale parameter indicating the speed of such transition to the value of \( c_f \), which is the final size of fixed costs of firing when \( L \) is

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\(^8\) In this paper, for the sake of simplicity, we specify adjustment costs as a function of gross employment changes, consistent with many papers in the literature [see, for example, Abowd and Kramarz (2003)]. For specifications based on both gross and net employment flows, see Hamermesh (1995).

\(^9\) Following the traditional setup, adjustment costs of capital are not introduced explicitly in the problem for ease of notation. Firms, however, can be thought of using both capital and labour. If capital adjustment costs are additively separable from those for labour, i.e. there are no interrelated adjustment costs, then one could still obtain the same first-order conditions for labour. In our model we also abstract from the choice of hours worked [see Chen and Funke (2004)].
greater than \( l \). By using this logistic function, changes in legislation can be accounted for in great detail. To see this, Figure 1 depicts the shape of the adjustment costs as a function of \( l \) and \( c_f \).

**Figure 1: The fixed cost function**

\[
f(L) = c_f \left( \frac{1}{1 + e^{-c_f(L-l)}} \right)
\]

A simple fixed cost of hiring, \( c_h \), is assumed for the hiring decision. Note that all parameters in equation (4) are assumed to be positive. Employment evolves according to

\[
\frac{dL_t}{dt} = M_t - \delta_l t,
\]

where \( \delta \) represents the deterministic quit rate. We assume that the multiplicative demand shock follows the geometrical Brownian motion,

\[
dZ_t = \eta Z_t dt + \sigma Z_t dW_t,
\]

where \( W_t \) is a standard Wiener process with independent, normally distributed increments, \( \eta \) is the deterministic drift parameter, and \( \sigma \) is the variance parameter.\(^{10}\)

The next task is to characterise the objective function of the firm. The firm chooses its optimal level of gross employment changes, \( M_t \), over time to maximise the intertemporal value of profits, subject to the employment stock accumulation [equation (5)] and the geometrical Brownian motion [equation (6)].

\(^{10}\) At this juncture an additional remark about this stochastic process is in place. A Brownian motion with a drift is the limit of a random walk with uneven probabilities for negative and positive changes. A positive (negative) drift implies that positive (negative) changes are more likely to occur than negative (positive) changes. The drift rate thus represents the bias of uncertainty.
More precisely, we assume that the firm maximises the present discounted value of its stream of current and expected future profits, defined as:

\[
V = \max_M E \left[ \int_{t=0}^{\infty} \left( A^{\psi_{\theta}} Z_t K^{\alpha_L} L_t^{\alpha_\varepsilon} - w L_t - C(M_t) \right) e^{-rt} dt \mid Z = Z_0, K = K_0, M = M_0 \right],
\]

s.t. (5) and (6),

where \( E[\cdot | \Omega_t] \) denotes the mathematical expectation given the information set available to the firm at period \( t \), \( \Omega_t \), \( r > 0 \) is the interest rate and \( wL_t \) is the wage bill. Applying Ito’s Lemma, the stochastic nature of this optimization problem requires the solution to the following Bellman equation:

\[
rV = \max_M E \left[ A^{\psi_{\theta}} ZK^{\alpha_L} L^{\alpha_\varepsilon} - wL - C(M) + V_L (M - \delta L) + \eta ZV_Z + \frac{1}{2} \sigma^2 Z^2 V_{ZZ} \right],
\]

where \( V \) represents the intertemporal value of the firm.\(^{11}\) Intuitively, equation (8) can be interpreted as follows: Should the option to hire be tradable and its risk diversifiable, then the expected value has to be equal to the foregone revenue from interest \((rV)\). The first-order conditions for gross employment changes yield

\[
\pm p_{h/f} + \gamma_{h/f} M = v,
\]

where \( v = V_L \). Note that the fixed costs of employment adjustment disappear in equation (9). However, the fixed costs of adjustment will enlarge the inaction area due to the fact that the firm only undertakes employment changes if a non-negative profit arises after deducting the fixed costs. It can be shown (see Appendix A) that hirings and firings occur when

\[
M \geq \frac{2\varepsilon}{\gamma_h} \geq 0
\]

and

\[
M \leq -\frac{2\varepsilon_{L}}{\gamma_f \left(1 + e^{\varepsilon_{L}}\right)} \leq 0.
\]

\(^{11}\) In the case of reversible hiring decisions, the effect of future profits does not occur because earlier hirings can be withdrawn at any time. Thus, it is sufficient to consider the marginal product of labour at present time \( t \) only.
The boundaries of the inaction area satisfy:

\begin{equation}
    v \geq p_h + \sqrt{2c_h \gamma_h} \quad \text{for hiring thresholds},
\end{equation}

and

\begin{equation}
    v \leq -p_f - \sqrt{\frac{2c_f \gamma_f}{1 + e^{-c_h(L-t)}}} \quad \text{for firing thresholds}.
\end{equation}

The upper threshold can be derived by finding the value of $v$ at which an additional worker generates non-negative profits. The lower threshold is found in a similar fashion. It is obvious that the higher the fixed costs of hiring/firing $c_h, c_f$, the greater is the number of hiring/firing in these employment decisions and the wider is the inaction area. The firm does not hire/fire employees for the boundaries of $v = \pm p_{hf}$; it waits until the numbers of hiring/firing reaching certain values to cover the non-trivial fixed costs of hiring/firing so that equations (9) are satisfied. The adjustment speed-related parameters $\gamma_h, \gamma_f$ also affect the numbers of hiring/firing. With a very small adjustment cost, the adjustment speeds increase and the firm tends to hire/fire more employees. The higher adjustment speeds due to smaller values of $\gamma_h, \gamma_f$ also imply that the values of $v$ do not deviate substantially from outside of $v = \pm p_{hf}$ – a smaller inaction area.  

For the levels of $M$ falling into the regime of $-\sqrt{2c_f \gamma_f} \left(1 + e^{-c_h(L-t)}\right) < M < \sqrt{2c_h \gamma_h}$, the firm does not hire or fire employees simply because the benefits from employment changes are not large enough to cover the fixed costs of hiring or firing, or even the proportional unit costs of hiring or firing. We can consider that $p_h + \sqrt{2c_h \gamma_h}$ as effective marginal hiring costs when considering mass-hiring, and $p_f + \sqrt{2c_f \gamma_f / \left(1 + e^{-c_h(L-t)}\right)}$ as effective firing costs.

The procedure in the Appendix A removes the nonlinear terms related to adjustment costs in Bellman equations and transfers them into parts of the effective hiring and firing costs. Thus, we have the following analytically solvable differential equation for the boundaries of (mass-) hiring/firing decisions:

\footnote{Note that contrary to the “now or never” decision in a traditional modelling framework with instantaneously and costlessly adjustable factors of production, the firm must choose the optimal time to fire or hire. This means that at every moment it must compare the continuation value, i.e., the value of the option when kept unexercised, and the value of an immediate firing or hiring decision.}
\[(r + \delta)v = \alpha_2 A^{1/\psi} ZK^{\alpha_1} L^{\alpha_2-1} - w - \delta v_L + \eta Z \nu + \frac{1}{2} \sigma^2 Z^2 \nu_{ZZ}, \]

where \(v_Z = V_{LZ}, \quad v_L = V_{LL}, \quad \text{and} \quad v_{ZZ} = V_{LZZ}. \) Equation (14), subject to the boundary conditions of equations (12) and (13), can be solved to obtain the hiring and firing thresholds \((Z_H \text{ and } Z_F)\) for the corresponding values of demand shocks.

After some algebra it can be shown (see Appendix B) that the particular solutions for \(v\) denoting the intertemporal marginal value of employees when no hiring and firing occurs takes the form

\[v^P = \frac{\alpha_2 A^{1/\psi} ZK^{\alpha_1} L^{\alpha_2-1}}{r + \alpha_2 \delta - \eta} - \frac{w}{r + \delta},\]

and the general solutions for \(v\) representing the value of the real options to hire and fire are denoted by

\[q^G = -B_1 \left( ZK^{\alpha_1} L^{\alpha_2-1} \right)^{\beta_1} + B_2 \left( ZK^{\alpha_1} L^{\alpha_2-1} \right)^{\beta_2},\]

where \(B_1\) and \(B_2\) are two unknown positive variables to be determined by the boundary conditions – the value-matching and smooth-pasting conditions – and \(\beta_1\) and \(\beta_2\) are the positive and negative characteristic roots of the following equation, respectively:

\[r + \delta + \delta \beta (\alpha_2 - 1) - \eta \beta - \frac{1}{2} \sigma^2 \beta (\beta - 1) = 0.\]

The term \(B_1 \left( ZK^{\alpha_1} L^{\alpha_2-1} \right)^{\beta_1}\) is usually interpreted as the real option to hire and the term \(B_2 \left( ZK^{\alpha_1} L^{\alpha_2-1} \right)^{\beta_2}\) is considered as the real option to fire. The value-matching and smooth-pasting conditions follow, and determine the thresholds of hiring and firing. Both conditions ensure that along the boundaries the firm is indifferent at the margin between an adjustment at date \(t\) and waiting \(dt\) to make the adjustment at date \(t + dt\). The value-matching conditions are:

\[
\frac{\alpha_2 A^{1/\psi} ZH K^{\alpha_1} L^{\alpha_2-1}}{r + \alpha_2 \delta - \eta} - \frac{w}{r + \delta} + B_2 \left( ZH K^{\alpha_1} L^{\alpha_2-1} \right)^{\beta_2} = p_h + \sqrt{2c_h y_h} + B_1 \left( ZH K^{\alpha_1} L^{\alpha_2-1} \right)^{\beta_1}.
\]

\[13\] The value-matching conditions involve the value function, while the smooth-pasting conditions concern its first-order derivatives.
and

\[
(19) \quad - \left[ \frac{\alpha^L A L^\alpha L^{\alpha-1}}{r + \alpha, \delta - \eta} + \frac{w}{r + \delta} \right] + B_1 \left( Z_F K^\alpha L^{\alpha-1} \right)^{\beta_1} = p_f + \sqrt{\frac{2c_f \gamma_f}{1 + e^{-c_s (l - \eta)}}} + B_2 \left( Z_F K^\alpha L^{\alpha-1} \right)^{\beta_2}.
\]

The smooth-pasting conditions take the forms:

\[
(20) \quad \frac{\alpha^L A L^\alpha L^{\alpha-1}}{r + \alpha, \delta - \eta} + \beta_1 B_1 Z_F^{\beta_1} \left( K^\alpha L^{\alpha-1} \right)^{\beta_1} = \beta_1 B_1 Z_F^{\beta_1} \left( K^\alpha L^{\alpha-1} \right)^{\beta_1}
\]

and

\[
(21) \quad - \left[ \frac{\alpha^L A L^\alpha L^{\alpha-1}}{r + \alpha, \delta - \eta} + \beta_1 B_1 Z_F^{\beta_1} \left( K^\alpha L^{\alpha-1} \right)^{\beta_1} = \beta_1 B_1 Z_F^{\beta_1} \left( K^\alpha L^{\alpha-1} \right)^{\beta_1}.
\]

Equations (18) - (21) consist of a non-linear system of four equations with four unknown variables, \( Z_F, K, B_1, \) and \( B_2. \) Generally, numerical methods have to be adopted because closed-form solutions cannot be derived. In order to develop a “feel” for the model and to “draw a map” of the labour demand sensitivity to various structural characteristics of the environment in which firms operate, we calibrate parameters as follows. We interpret periods as years and annual rates are used where applicable. Where possible, parameter values are drawn from empirical studies.\(^{14}\) Our base parameters are \( \sigma = 0.1, \eta = 0, r = 0.05, \psi = 0.05, \alpha = 0.7, \beta = 1.5 \) and \( A = 1. \) In practice, measuring product market competition is a complex task. In our baseline parameter specification the price elasticity of demand parameter is set at \( \psi = 1.50 \) as in Bovenberg et al. (1998). The deterministic drift term \( \eta \) has been set to zero to avoid any “bias in uncertainty”. The labour share \( 1-\alpha \) (profit share \( \alpha \)) is 0.7 (0.3). For simplicity, we normalise capital such that \( K = 6.0 \). This does not affect the qualitative results. We set \( A = 1 \) without loss of generality. The baseline threshold level for application of dismissal protection is \( l = 5 \). The choice of the remaining labour adjustment cost parameters can be explained as follows. Beyond the threshold \( l = 5, \)

\(^{14}\) It should be acknowledged that despite efforts to rely on multiple sources and datasets, there is inevitably an arbitrary and subjective aspect to some dimensions of the calibration. In particular, it is difficult to ascertain and quantify the extent of enforcement of statutory restrictions across firm sizes. We suggest taking an eclectic approach to capturing key economic features of policy interest. The basic idea is to choose coefficients that seem reasonable based on economic principles, available econometric evidence, and an understanding of the functioning of the economy, and then to look at how sensible the responses of the real options model are.
the effective firing costs should reach 0.6.\textsuperscript{15} Our benchmark value of \( p_f + \frac{2c_f \gamma_f}{\sqrt{1+e^{-c_0(l-I)}}} \) beyond \( l = 5 \) is \( p_f + \sqrt{2c_f \gamma_f} = 0.05 + \sqrt{2 \times 0.3 \times 0.5} = 0.5977 = 0.6 \); the effective hiring costs \( p_h + \sqrt{2c_h \gamma_h} = 0.02 + \sqrt{2 \times 0.06 \times 0.01} = 0.0546 \) are also in the range of 0.06 as suggested by Bentolila and Bertola (1990). The corresponding hiring and firing \( M \)'s are \( M_h = \frac{2c_h}{\gamma_h} = 3.46410 \) for hiring and \( M = -\frac{2c_f}{\gamma_f (1+e^{-c_0(l-I)})} = -1.09545 \) for firing after \( L > l = 5 \). Figure 2 and 3 provide a graphical description of the pattern of employment adjustment for \( l < 5 \) vs. \( l < 10 \).

\textbf{Figure 2: The Effects of Dismissal Protection Regulation with Exempted Establishment \( l < 5 \)}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure2.png}
\caption{The Effects of Dismissal Protection Regulation with Exempted Establishment \( l < 5 \)}
\end{figure}

\textbf{Figure 3: The Effects of Dismissal Protection Regulation with Exempted Establishment \( l < 10 \)}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure3.png}
\caption{The Effects of Dismissal Protection Regulation with Exempted Establishment \( l < 10 \)}
\end{figure}

\textsuperscript{15} Our parameters convey the message in Bentolila and Bertola (1990). Their estimated firing costs for Germany are in the range of \( 0.562 \leq p_f \leq 0.750 \), and their hiring cost estimate (excluding on-the-job-training) for Germany is 0.066 of the average annual wage.
The intuitive graphs dichotomize the space spanned by $Z$ shocks into action and inaction areas. In the inaction area the marginal reward for changing employment is insufficient: neither hiring nor firing is optimal. The comparison of Figure 2 and Figure 3 reveals what is happening when countries try to deregulate labour markets by shifting the threshold where dismissal protection will be effective from $l = 5$ to $l = 10$. The widening of the inaction area beyond the threshold indicates that the “Protection Against Dismissal Act” reduces the propensity to hire and fire with respect to the unregulated world.\(^\text{16}\) The direct cost of employment protection makes adjustment of labour more expensive, which tends to lower firms’ willingness to hire. On the other hand, effective legal protection of existing employment relationships lowers the occurrence of firing during recessions. As firing and hiring incentives work in opposite directions, the impact of tighter or softened adjustment costs for labour is theoretically ambiguous. Bentolila and Bertola (1990) and Bertola (1992) have demonstrated in real option models that the overall impact depends, inter alia, upon the size of the adjustment costs, the functional forms and the discount rate. Our numerical findings verify the conjecture that the overall effect is a reduction of the speed of adjustment to shocks, but for fixed wages $w = \bar{w}$ the net effect turns out to be mostly positive. In other words, the simulations imply that firing costs have more of an effect on the firing decision than on the hiring decision, thereby increasing long-term employment.\(^\text{17}\)

Focusing on the results close to the thresholds, the calibration results indicate that the anticipation of future firing costs may have current effects for a hiring firm even when the more stringent firing regime beyond the threshold is absent at the time of decision making. Elaborating on this idea and using our formal theoretical model of labour demand decisions under uncertainty, the results in Figure 2 and 3 indicate that latent legal constraints can affect firms’ employment policy even when these firing constraints are currently slack. The numerical calibrations elegantly demonstrate this, as the outcome of a forward-looking behaviour by the small firm that expects future legal constraints to bind, resulting in current employment decisions to be a function both of the current legal framework but also expectations about their more stringent future path after growing beyond the threshold.

3. Further Robustness

The main conclusion of the previous Section is that firing costs beyond some thresholds tend to reduce both dismissals and hirings. Its overall impact on aggregate employment is likely to be positive. This notwithstanding, the effects of employment protection are likely to be different across firm size. We are aware of the many caveats that such ceteris-paribus comparison can arise. A first concern is that tighter employment regulations may diminish company’s ability to cope with a rapidly changing

\(^{16}\) The widening of the gap is consistent with the empirical evidence presented in Bauer et al. (2004) analysing worker flows in German establishments from March 1995 to March 1998 who have undergone periods of protection and non-protection as a consequence of the repeated changes in the German dismissal legislation.

\(^{17}\) Pissarides (2001) has outlined another mechanism. He has shown that dismissal protection might increase welfare by providing insurance against unemployment.
environment driven by globalisation, technical progress, and organisational change. Caballero et al. (2004) have demonstrated that job security legislation hampers the creative-destruction process and lowers productivity growth. The clear and robust result is that tight job security regulations lower annual productivity growth somewhere between 0.8 and 1.2 percent. Samaniego (2006) has demonstrated in a theoretical model that high firing costs slow the diffusion of new technologies via the mobility of entrepreneurial resources. Therefore firing costs are particularly detrimental in industries in which the rate of technical progress is rapid. Finally, Acemoglu et al. (2006) and Colecchia (2002) have also demonstrated in an independent literature that a more competitive institutional setting will contribute to a more innovative and dynamic economy through thriving entrepreneurial activity. Although these studies did not use the same methodology as in this paper, their results which are based on theoretical models and regressions reveal a similar story as the one below.

In order to gauge the costs of job security provisions with endogenous creative destruction processes, we adapt our baseline real options model and assume that the level of aggregate productivity $A$ is a function of total effective employment adjustment costs (TEEA), i.e. the magnitude of the inaction area:

\[(22) \quad A = \frac{1}{1 + A_0 \times TEEA},\]

where $TEEA = p_h + \sqrt{2c_h^2 \gamma_h} + p_f + \sqrt{2c_f^2 \gamma_f / \left[1 + e^{-c_0(L-l)}\right]}$.

<table>
<thead>
<tr>
<th>$A_0$</th>
<th>$L &lt; 10$</th>
<th>$L \geq 10$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.08</td>
<td>0.9917</td>
<td>0.9504</td>
</tr>
<tr>
<td>0.16</td>
<td>0.9835</td>
<td>0.9055</td>
</tr>
<tr>
<td>0.24</td>
<td>0.9755</td>
<td>0.8646</td>
</tr>
</tbody>
</table>

The implications of employment-protection-induced productivity changes upon the inaction area for $l = 10$ are summarised graphically in Figure 4 for $A_0 = 0.08$, $A_0 = 0.16$, and $A_0 = 0.24$, respectively.\(^{18}\) Taking this productivity impact into account implies that contrary to the baseline model labour market regulations could indeed be a barrier to employment.

\[^{18}\text{In the simulations we assume that the extra gains in productivity will not be eroded by higher wages. The deregulated labour market and the threat that jobs in advanced economies could move abroad may help to hold wages down.}\]
A second concern is that wages in the baseline model have been fixed although much of the debate of persistent unemployment in European economies has focused on the wage formation systems. Caballero and Hammour (1998) have shown that dismissal protection legislation like any other mandatory employment protection measure creates a hold-up problem enabling insiders (incumbent workers) to bid up wages once they are employed.\textsuperscript{19} In other words, firing costs make it difficult for firms to fire workers, so firms hesitate to hire them in the first place, strengthening the hand of unions bargaining with firms to set a wage. Contrary to the traditional literature holding wages fixed and looking at the employment effect of different degrees of job security provisions, we therefore adapt our model to an insider-outsider mechanism where firing costs increase the bargaining power of incumbent workers [see, e.g., Díaz-Vázquez and Snower (2003) and Lindbeck and Snower (1988)].\textsuperscript{20} Explicitly modelling the endogenous response of heterogeneous firms will help to deepen our understanding of how firms, industries and economies respond to policy reforms such as deregulation. For simplicity and for clarity of exposition we assume that wages are determined as

\begin{equation}
(23) \quad w = 1 + (r + \delta)(EFC),
\end{equation}

where \(EFC\) denotes effective firing costs depicted in

\begin{equation}
\frac{p_f + \sqrt{2c_f Y_f f \left(1 + e^{-v_0(l_t - t)}\right)}}{1 + e^{-v_0(l_t - t)}}.
\end{equation}

Equation (20) constitutes an additional “wage mechanism” in the regulation transmission channel and can be seen as encompassing various sources of wage rigidity. While being a short cut to a strictly micro founded wage equation, equation (23) constitutes a plausible starting point for analysing the impact of endogenous wages on the regulation transmission process. Figure 5 reports the numerical results of this experiment for \(A_0 = 0.08\), \(A_0 = 0.16\), and \(A_0 = 0.24\), respectively.

\textsuperscript{19} In countries with higher firing costs a large share of workers with fixed-term contracts tend to insulate insiders (permanent workers) from adjustment, thereby increasing their bargaining power.

\textsuperscript{20} Lazear (1990) has claimed that the non-wage labour costs arising from mandatory dismissal protection will be offset by an efficient contract or bargaining process (in the sense that they do not influence equilibrium
Adjustment costs of labour again induce firms to hoard labour during recessions, and also to hire fewer workers during boom periods. Contrary to the baseline model, however, the shape of the firing cost profiles implies that the insider-outsider considerations provide a channel through which the impact of firing costs is pulled, via wages, towards a negative impact on average employment.\textsuperscript{21}

In this Section we have provided some tests of the sensitivity and robustness of the baseline model. Taken together, the augmented models follows a more pronounced deregulatory line. Of course, the endogenous productivity and wage effects are hard to quantify. Nevertheless, this paper shows that they are important channels through which employment protection might affect macroeconomic aggregates and therefore they are the ones that should be the focus of attention.

4. Conclusions

Attitudes and policies towards deregulation of labour markets have been subject to considerable controversy and flux. Our paper fits neatly into this debate, and provides some fresh evidence on labour demand dynamics associated with asymmetric job security provisions across the firm size distribution. There has been considerable debate among politicians, unions, employer associations and economists about dismissal protection legislation. To contribute to this debate, we have designed and presented an economically meaningful and transparent dynamic model characterizing the firm’s optimal behaviour under uncertainty. While highly stylised, the real options model singles out important transmission channels and allows policy-makers to study the implications of policy interventions in alternative model specifications.

\textsuperscript{21} This result confirms the „all or nothing“ warning issued by Coe and Snower (1997) and Orszag and Snower (1998). They argue that piecemeal labour market reforms may have had so little success because they disregarded the complementarities between a broad range of policies and institutions.
Appendix A: The Boundaries of the Inaction Area

By substituting (9) in the text back into the Bellman equation (8) in the text and rearranging we obtain for the hiring and firing decisions:

\[
(A1) \quad rV = A^{1/p} ZK^{\alpha_1} L^{\alpha_2} - wL - c_h + \frac{1}{2} \left( v - \frac{p_h}{\gamma_h} \right)^2 - \delta v L + \eta Z V + \frac{1}{2} \sigma^2 Z^2 V_{zz};
\]

\[
(A2) \quad rV = A^{1/p} ZK^{\alpha_1} L^{\alpha_2} - wL - \frac{c_f}{1 + e^{-\tau_0(t_t-t)}} + \frac{1}{2} \left( v + \frac{p_f}{\gamma_f} \right)^2 - \delta v L + \eta Z V + \frac{1}{2} \sigma^2 Z^2 V_{zz}.
\]

The firm would hire/hire marginal employees only if the total revenue net costs of hiring/hiring are non-negative. Thus, for hiring decision \( (M \geq 0) \), the firm has benefit of hiring \( M \) employees – the value of the firm increases by \( Mv \); for hiring those \( M \) employees, the firm pays the total cost of employment for hiring. The hiring decisions would only happen for a certain \( M \) or greater as long as the following equation is satisfied:

\[
(A3) \quad Mv - \left( c_h + p_h M + \frac{1}{2} \gamma_h M^2 \right) \geq 0.
\]

In economic downturns, the firm endures a loss so that the value of \( v \) is negative. By firing \( M \) employees \( (M \leq 0) \), the loss of the firm is reduced by \( Mv \), which is considered to be the benefit of firing \( M \) employees; the firing also incurs some total cost of adjustment. The firm only fire a certain number of employees or more if the following relationship is satisfied:

\[
(A4) \quad Mv - \left( \frac{c_f}{1 + e^{-\tau_0(t_t-t)}} - p_f M + \frac{1}{2} \gamma_f M^2 \right) \geq 0.
\]

Multiplying both sides of (9) in the text by \( M \) and substituting into (A3) and (A4) gives

\[
M^2 \geq \frac{2c_h}{\gamma_h} \quad \text{for hiring},
\]

and

\[
M^2 \geq \frac{2c_f}{\gamma_f \left( 1 + c_v e^{-\tau} \right)} \quad \text{for firing}.
\]

Thus, for (mass-) hiring starting thresholds, we shall have

\[
(A5) \quad M \geq \sqrt{\frac{2c_h}{\gamma_h}} > 0;
\]

and for (mass-) firing starting thresholds, we need the following relationship

\[
(A6) \quad M \leq -\sqrt{\frac{2c_f}{\gamma_f \left( 1 + c_v e^{-\tau} \right)}} < 0.
\]
Substituting (A5) and (A6) back into equation (9) in the text respectively gives the hiring/firing regimes for the intertemporal marginal value of the firm

\[ v \geq p_h + \sqrt{2c_h \gamma_h} \] for hiring regime,

and

\[ v \leq -p_f - \frac{2c_f \gamma_f}{\sqrt{1 + e^{-\alpha_1(L)}}} \] for firing regime.

The boundaries of the inaction area or the beginning points of hiring and firing regimes, where equations hold, are then determined by the following two equations.

(A7) \[ v = p_h + \sqrt{2c_h \gamma_h} \] for hiring thresholds,

and

(A8) \[ v = -p_f - \frac{2c_f \gamma_f}{\sqrt{1 + e^{-\alpha_1(L)}}} \] for firing thresholds.

Substituting (A7) and (A8) back into Bellman equations (A1) and (A2) gives the following unified differential equations for hiring and firing:

(A9) \[ rV = A^{1/p} ZK^\alpha L^{\alpha z} - wL + \delta vL + \eta ZV_Z + \frac{1}{2} \sigma^2 Z^2 V_{ZZ} \]

Using the definitions \( v = V_L, \ v_Z = V_{LZ}, \ v_L = V_{LL}, \) and \( v_{ZZ} = V_{LZZ} \) and differentiating both sides of equation (A9) with respect to \( L \) yields

(A10) \[ (r + \delta) v = \alpha_z A^{1/p} ZK^\alpha L^{\alpha z-1} - w - \delta vL + \eta Zv_Z + 1 \frac{1}{2} \sigma^2 Z^2 v_{ZZ} \]

which is equation (14) in the text.

Appendix B: The Particular and General Solutions for \( v \)

**Particular solutions**

Assume that the particular solutions have the following functional form:

(B1) \[ v^p = aZK^\alpha L^{\alpha z-1} + b, \]

We then have the following relationships:

(B2) \[ \eta Zv_Z = a \eta ZK^\alpha L^{\alpha z-1}, \]

(B3) \[ v_{ZZ} = 0, \]
(B4) \(-\partial_{L} v_{L} = -a \delta (\alpha_{2} - 1) ZK^{\alpha_{2}} L^{\alpha_{2} - 1}.\)

Substituting into equation (14) in the text gives:

(B5) \[a(r + \alpha_{2} \delta - \eta) - \alpha_{2} A^{1/p} \] \[ZK^{\alpha_{2}} L^{\alpha_{2} - 1} + [(r + \delta) b + w] = 0.\]

The above equation should hold for any value of marginal product of employees. Thus, we have

(B6) \[a = -\frac{\alpha_{2} A^{1/p}}{r + \alpha_{2} \delta - \eta},\]

(B7) \[b = -\frac{w}{r + \delta},\]

which yields the particular solution (15) in the text.

**Homogenous solutions**

The homogenous part of equation (14) in the text is represented by

(B6) \[(r + \delta) v = -\delta v_{L} + \eta Z v_{z} + \frac{1}{2} \sigma^{2} Z^{2} v_{zz}.\]

The homogenous solutions should have the same components as in particular solutions. Therefore, assume the following functional form for homogenous solutions:

(B7) \[v^{\beta} = B (ZK^{\alpha_{2}} L^{\alpha_{2} - 1})^{\beta},\]

where \(A\) is constant and to be determined by value-matching and smooth-pasting conditions. We then have the following relationships for homogenous solutions:

(B8) \[\eta Z v_{z} = \eta \beta B (ZK^{\alpha_{2}} L^{\alpha_{2} - 1})^{\beta},\]

(B9) \[\frac{1}{2} \sigma^{2} Z^{2} v_{zz} = \frac{1}{2} \sigma^{2} \beta (\beta - 1) B (ZK^{\alpha_{2}} L^{\alpha_{2} - 1})^{\beta},\]

(B10) \[-\partial_{L} v_{L} = -\delta (\alpha_{2} - 1) \beta B (ZK^{\alpha_{2}} L^{\alpha_{2} - 1})^{\beta}.\]

Substituting into equation (B6) and rearranging gives:

(B11) \[r + \delta + \delta \beta (\alpha_{2} - 1) - \eta \beta - \frac{1}{2} \sigma^{2} \beta (\beta - 1) = 0,\]

which is (17) in the text. There are two characteristic roots for \(\beta\): one positive and one negative: \(\beta_{1} > 0 > \beta_{2}\). Therefore, the homogenous (general) solutions are shown as follows:

(B12) \[v^{\beta} = -B_{1} (ZK^{\alpha_{2}} L^{\alpha_{2} - 1})^{\beta_{1}} + B_{2} (ZK^{\alpha_{2}} L^{\alpha_{2} - 1})^{\beta_{2}},\]
which corresponds to real options to hire and fire employees respectively, and is equation (16) in the text.
References:


