Working Time and Employment under Uncertainty

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Abstract
The standard literature on working time has modelled the decisions of firms in a deterministic framework in which firms can choose between employment and overtime (given mandated standard hours). Contrary to this approach, we consider the impact of uncertainty and real options on the decision of working time, i.e. we examine the determinants of employment and hours in a stochastic framework. We conclude the theoretical analysis with a number of simulation exercises to illustrate the working of the model.

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I. Introduction

The persistence of high unemployment, and the rise of non-employment for some groups such as older males, in various European countries has reignited academic and political debate over work sharing by legislated reduction of the standard working week. Thus a 35 hour week has been enacted in France for firms with over 20 employees, and other firms must follow suit within two years. A similar legislation is pending in Italy [OECD (1998)]. In Germany, the largest union for the metal working industry (IG Metall) has recently even asked for a 32 hour week agreement. Similar programs are envisaged in other European countries, which hope that hours reductions will be an efficient policy for reducing unemployment. The basic intuition underlying these suggestions is that reducing standard hours per employee would encourage firms to hire additional employees as they seek to replace the labour services currently provided by those employees putting in longer hours. Such measures remain controversial, and the earlier studies of employment effects of working time reduction summarized by Hart (1987) and Calmfors and Hoel (1988, 1989) were inconclusive but generally sceptical.¹ On the other hand, a number of more recent theoretical studies by Houpis (1993), and Contensou and Vranceanu (2000), FitzRoy et al. (2002), and Marimon and Zilibotti (2000) tend to be somewhat more optimistic, while the OECD (1998) has emphasized the importance of measures that would complement working time reduction to encourage job creation.

A common feature of these studies on working time is that the decisions of firms are modelled in a deterministic framework in which firms can choose between employment and overtime (given mandated standard hours). An exogenous reduction of standard hours then leads to an ambiguous employment effect depending upon the overtime premium and other parameters of the model. However, these traditional approaches do not consider the impact of uncertainty and real options on the decision of working time. Given these employment policy debates, the paper tries to consider one of the most recent developments of the literature on investment theory to model the determinants of employment and hours of work. In other words, we would like to use the theory of real options to model the use of overtime adjustment in relation to employment adjustment.

The general idea behind the so-called theory of real options is that each investment project can be assimilated, in its nature, to the purchase of a financial call option, where the investor pays a premium price in order to get the right to buy an asset for some time at a predetermined price (exercise price), and eventually different from the spot market price of the asset (strike price).² Analogously, the firm, in its investment decision, pays a price (the cost of setting up the project) which gives her the right to use the capital (exercise price), now or in the future, in return for an asset worth a strike price. Taking into account this approach, the calculus of profitability of each single

¹ A thorough review of the literature can be found in OECD (1998), pp. 117-148. Many labor economists have derided the idea of the 35-hour work week to reduce unemployment as the "lump-of-labour fallacy".
² The use of real option theory to analyse factor demand decisions under uncertainty has become increasingly popular. See, for example, Dixit and Pindyck (1994) and Coy (1999).
investment project cannot be done simply applying the net present value rule to the expected future cash flows of the operation, but has rather to consider the following three characteristics of the investment decision:

1. there is *uncertainty* about future rewards from the investment;
2. there is some leeway about the *timing* of the investment and
3. the investment is partially or complete *irreversible*.

The first characteristic of the investment decision derives from the fact that the investors have no perfect information. As a result they form expectations and beliefs on the future behaviour of the economic variables which cannot be predicted with certainty. The second characteristic is directly related to the uncertainty: investors might want to postpone their investment from period $t$ to period $t+1$ in order to get more information, refine their beliefs and reduce uncertainty. This of course entails an opportunity cost of waiting in terms of missed opportunities, should the economic variables in period $t+1$ be such that the investor would have made profits had the investment been undertaken at time $t$. Finally, the investor has to take into account the fact that the initial cost of the investment is at least partially sunk, i.e. he or she cannot recover it all should he or she change his mind after the investment has been undertaken. As a result, the weight of the uncertainty in the determination of the net present value is higher the higher is the sunk cost of the investment. The novel aspect of this paper is to apply the real option theory to the case of employment and hours determination, i.e. we intend to model overtime hours combined with firing and hiring costs as a rational response of firms to an uncertain environment. In particular, we distinguish between altering labour input along the extensive and intensive margins, and accordingly decompose labour input into an employment decision – the extensive margin – and hours worked per worker – the intensive margin. This takes a step towards providing a satisfactory framework which can be used to analyse and clarify some of the policy debates and empirical regularities evidenced above.\(^3\)

The remainder of this paper is organised as follows. Section II begins by laying out the basis analytical framework, and show how employment policies and timing can be treated as an optimal stopping problem. The results from model calibrations are presented in the succeeding section III. We conclude in section IV by discussing the implications of our results for the worksharing and employment protection debate and offer suggestions for future research.

\(^3\) In related work, Bentolila and Bertola (1990), Bertola (1990), Bell (1996), Chen and Zoega (1999) and Booth et al. (2001) have used similar modelling frameworks. None of these papers, however, has focused on standard hours and overtime. Our approach will therefore reduce the bias that may arise through failure to control for the endogeneity of overtime hours.
II. The Theory of the Firm

A. The Cobb-Douglas Case

In order to get at the basic issues and obtain results that are reasonably easy to interpret, we introduce a real options model in a simple way as possible. The main objective in specifying the technology is to model the basic trade-offs which firms are faces in deciding upon the working hours of its employees, and the number of employees to hire. For the sake of analytical convenience, the production function of the representative firm in terms of value added is denoted by the Cobb-Douglas production function

\[ Y = K^\alpha (Ng(H))^{1-\alpha}, \]

where \( H \) actual hours, \( N \) employment level, \( x \) the fixed costs of employment, \( H_s \) is standard contract hours. The capital stock \( K \) is taken as given at any point in time, giving rise to strict concavity of the production function. We also abstract from changes in the utilisation of capital. It is still noteworthy that economic theory has not still reached an unanimous consensus on the sign of the relationship between overtime hours worked and uncertainty. While it seems natural for the average person on the street to think that higher demand uncertainty means less employment and more overtime, results from theoretical models critically depend upon hypotheses made regarding agent’s preferences, market regimes, and the type of production technology adopted by firms. It is well known that within standard factor demand models factor demand and uncertainty are positively correlated. This result depends upon the convexity of the firm’s profit function with respect to the stochastic variable, usually the output price. Caballero (1991) has demonstrated that irreversibility by itself (or asymmetric adjustment costs) is not sufficient in order to obtain a negative relationship between factor demand and uncertainty. Other hypotheses on the relationship between current and future employment should also be considered as, for instance, the presence of imperfect competition or decreasing returns to scale. The positive relationship between factor demand and price uncertainty is at odds with what seems to occur in the real world, where the media often report the concern of entrepreneurs and public authorities of the negative effects of uncertainty on project returns and, therefore, on the willingness to expand capacity. In this section and the next, we therefore allow for imperfect competition, i.e. we assume that the firm faces an isoelastic demand function where \( p \) and \( Y \) denote the price and the output, respectively [See Abel and Eberly (1994)].

\[ p = Y^{(1-\psi)/\nu} Z, \quad \psi \geq 1, \]
where $Z$ denotes the demand shock, and $\psi$ is an elasticity parameter that takes its minimum value of 1 under perfect competition. Therefore, current profits, measured in units of output, are defined as,

$$\Pi = Z^{\alpha_1} \left( Ng (H) \right)^{\alpha_2} - \left[ w(H) + x \right] N$$

where $\alpha_1 = \alpha / \psi$ and $\alpha_2 = (1 - \alpha) / \psi$, $w$ hourly wages, and $x$ the fixed costs of employment. It is important to note that $x$ (for example, work space for the workers) is interpreted as a flow in (4). The existence of fixed costs per worker $x$ tends to make firms want higher hours in order to spread these costs over more hours of work.\(^5\) Risk-neutral firm chooses actual hours and employment to maximise its expected discounted value of profits. The firm’s expected value of discounted profits without any firing and/or hiring costs is

$$V = \max_{N, H} \int_0^\infty \left[ Z^{\alpha_1} \left( Ng (H) \right)^{\alpha_2} - \left[ w(H) + x \right] N \right] e^{-rs} ds,$$

where $r$ is the real rate of interest. According to equation (4), firms choose how many people to employ, and the specific number of hours, given the wage schedule. It is assumed that workers never quit and the demand factor $Z$ follows the geometric Brownian motion

$$dZ = \eta Z dt + \sigma Z d\mathcal{W}$$

where $d\mathcal{W}$ is a Wiener process; $d\mathcal{W} = \sqrt{dt}$ (since $\mathcal{W}$ is a normally distributed random variable with mean zero and a standard deviation of unity), $\eta$ is the drift parameter and $\sigma$ the variance parameter. Thus, we have an optimal stopping problem – we must determine when it is optimal to hire or fire workers, given the stochastic evolution of $Z$. Using Itô’s Lemma, the Bellman equation for the value $V$ at time zero, in the continuation region is

$$rV = \max_{N, H} \left( Z^{\alpha_1} \left( Ng (H) \right)^{\alpha_2} - \left[ w(H) + x \right] N + \eta Z V_z + \frac{1}{2} \sigma^2 Z^2 V_{zz} \right)$$

The first term on the right-hand side is revenue, $\left[ w(H) + x \right] N$ is the employment-related bill, $\eta Z V_z$ is the gain due to demand growth, and the last term is the change in the value of the firm caused by changes in demand. The first-order conditions for $H$ is:

\(^4\) See Zeira (1987) on the role played by imperfectly competitive markets.

\(^5\) We consider the hourly wage to be exogenous because union models such as those of Calmfors (1985) indicate that the direction of the wage change is ambiguous when standard hours are cut. Another reason is the
(7) \[ \alpha_2 ZK^{\alpha_1} N^{\alpha_2} g^{\alpha_1-1}(H)g'(H) - w'(H)N = 0 \]

After solving equation (7), the variable \( H \) become a function of \( Z \) given the functions of \( w \) and \( g \). Note that since \( Z \) follows a stochastic process, the values of \( Z \) in hiring and firing decisions should be different in (7). To find the optimal condition for employees with the existence of firing costs and hiring costs, we need to obtain the value of the marginal employed worker first \((v = V_N)\) and then compare the marginal value of employees with the marginal hiring costs and firing costs. We take the derivative of (6) with respect to \( N \)

(8) \[ rv = \alpha_2 ZK^{\alpha_1} N^{\alpha_2} g^{\alpha_1-1}(H) - [w(H) + x] + \eta Zv_x + \frac{1}{2} \sigma^2 Z^2 v_{zz} \]

where \( v = V_N \) is the value of employing the marginal worker. The solution for \( v(Z) \) consists of the particular integral and the complementary function. We first deal with identification of uncertainty effects in the very special case where hiring and firing costs are zero. This special case turns out to be useful as a starting point and for comparisons. Then we turn to the general case with positive hiring and firing costs. In the absence of hiring and firing costs, the particular integral may be expressed as

(9) \[ v^p(Z) = E\left[ \alpha_2 ZK^{\alpha_1} N^{\alpha_2} g^{\alpha_1-1}(H) - [w(H) + z] \right] e^{-rs} ds \]

which is the expected present value of the marginal employed worker. This integral can be rewritten as

(10) \[ v^p(Z) = \frac{\alpha_2 ZK^{\alpha_1} N^{\alpha_2} g^{\alpha_1-1}(H)}{r - \eta} - \frac{w(H) + z}{r} \]

The firm’s option value of hiring in the future and its option value of firing once the worker is employed are measured by the complementary function:

(11) \[ rv = \eta Zv_x + \frac{1}{2} \sigma^2 Z^2 v_{zz} \]

existing evidence that unions involved in worksharing have successfully campaigned for increases in the hourly wage to "compensate" for the hours lost.
Letting \( v^G \) be the value of the option, the general solutions for the hiring and firing options have the following forms, respectively,

\[
(12) \quad v^G_T(Z) = A_1 Z^{\beta_1}
\]

and

\[
(13) \quad v^G_F(Z) = A_2 Z^{\beta_2}
\]

where \( \beta_1 \) and \( \beta_2 \) are the positive and negative roots of the following characteristic equation:

\[
(14) \quad \frac{1}{2} \sigma^2 \beta (\beta - 1) + \eta \beta - r = 0
\]

To satisfy the boundary conditions that \( v^G_T(0) = 0 \) and \( v^G_F(\infty) = 0 \), we use the positive solution for \( v^G_H \) and the negative solution for \( v^G_F \).

We now add fixed hiring \( (T) \) and firing \( (F) \) costs to the model with both \( T \) and \( F \) being payable by the firm.\(^6\) When there are fixed costs of either hiring or firing, the firm will consider the option value of maintaining her current position against the alternative of hiring or firing. The value of the marginal, employed worker is equal to the sum of \( v^P \) and \( v^G \) in the continuation region. In order to derive the two thresholds for hiring and firing, we then compare the value of the worker to the direct and indirect costs of hiring (firing) the workers. The definitions of the hiring and firing barriers, \( Z_T \) and \( Z_F \), are given by the value-matching and smooth-pasting conditions below. It is straightforward to show that according to the value-matching conditions the firm would find it optimal to exercise its option to hire or fire the marginal worker once \( Z \) hits one of the two barriers:

\[
(15) \quad \frac{\alpha_2 Z_T K^{N^T} N^{\alpha_2 - 1} g^{\alpha_3} (H)}{r - \eta} - w(H) + \frac{x}{r} + A_2 Z_T^{\beta_2} = T + A_1 Z_T^{\beta_1},
\]

and

\[
(16) \quad - \left[ \frac{\alpha_2 Z_F K^{N^F} N^{\alpha_2 - 1} g^{\alpha_3} (H)}{r - \eta} - w(H) + \frac{x}{r} \right] + A_2 Z_F^{\beta_2} = F + A_1 Z_F^{\beta_1}
\]

\(^6\) \( T \) can be thought of as representing the screening and training costs associated with the recruitment of a new employee and \( F \) as the severance costs imposed by legislation when dismissing an employee.
The left-hand sides of (15) and (16) show the marginal benefit from hiring/firing a worker and the right-hand sides the corresponding marginal costs. The marginal benefit of hiring a worker is equal to the sum of the present discounted value of his productivity net of wages and the value of the option to fire him. The firm’s ability to fire raises the benefit from employing a worker. The marginal cost of hiring is the sum of the direct hiring costs and the sacrificed option to hire him in the future. By hiring a worker today, the opportunity to do so in the future – when conditions may be more favourable – is sacrificed. Similarly, by firing a worker, the opportunity to do so in the future – when demand conditions may be even more adverse – is sacrificed, and the opportunity to hire him again is gained. The value of the two options depends on expectations about changes in demand. The option to hire is valuable if firms expect demand to increase in the future, while the option to fire is the more important if they expect it to fall. The smooth-pasting conditions ensure that hiring (firing) is not optimal either before nor after the hiring (firing) threshold is reached. In technical terms, this means:

\[
\frac{\alpha_2 K^{\alpha_2} N^{\alpha_2-1} g^{\alpha_2}(H)}{r-\eta} + A_2 \beta_2 Z_T^{\beta_2-1} = A_1 \beta_1 Z_T^{\beta_1-1}
\]

and

\[
-\frac{\alpha_2 K^{\alpha_2} N^{\alpha_2-1} g^{\alpha_2}(H)}{r-\eta} + A_1 \beta_1 Z_F^{\beta_1-1} = A_2 \beta_2 Z_F^{\beta_2-1}.
\]

Equations (15) - (18) form a non-linear system of equations with four unknown parameters, \(Z_T, Z_F, A_1, \text{ and } A_2\), and can be solved for numerically once the solutions for \(\beta_1\) and \(\beta_2\) are obtained from (14) and optimal values for \(H\) are found for the the values of \(Z_T\) and \(Z_F\) via equation (7):

\[
(1-\alpha)Z_T K^{\alpha} N^{1-\alpha} g^{-\alpha}(H)g'(H) - w'(H) N = 0
\]

and

\[
(1-\alpha)Z_F K^{\alpha} N^{1-\alpha} g^{-\alpha}(H)g'(H) - w'(H) N = 0.
\]

In order to calculate the thresholds for hiring \((Z_T)\) and firing \((Z_F)\) a marginal worker, we have to select a functional form for \(g(H)\) and \(w(H)\). Following Hart (1987) and Santamäki (1984) we model
labor services as a piece-wise continuous and nonlinear function of mandated standard hours $H_s$ and actual hours worked $H$:

$$\begin{align*}
\text{if } & H > H_s \\
\text{then } & g(H) = \frac{H}{H_s} \\
\text{if } & H \leq H_s \\
\text{then } & g(H) = \frac{1}{H} 
\end{align*}$$

Standard contract hours are exogenously given to the firm which determines actual hours as a control variable besides employment. Following the literature we assume that $0 < \delta < 1$ so that $g(H)$ is strictly concave and the problem of the firm is well defined. An exogenous reduction of $H_s$ may increase or decrease $g(H)$ and – depending on the overtime wage premium – increase or decrease employment and labor services. Following Hart (1987) we introduce a piece-wise linear wage equation

$$\begin{align*}
\text{if } & H > H_s \\
\text{then } & w(H) = w_sH + \alpha w_s(H - H_s) \\
\text{if } & H \leq H_s \\
\text{then } & w(H) = \frac{w_s}{H_s} 
\end{align*}$$

according to which firms pay a constant premium $\alpha > 1$ on overtime hours $(H - H_s)$. In other words, $\alpha$ is the legally determined multiple of the standard wage $w_s$ paid for regular hours. Equations (3) and (22) imply that we allow for quasi-fixed labour costs and wage schedules that are increasing in hours worked. The corresponding thresholds for hours ($H_T$ and $H_F$, respectively) can be calculated in a similar way. It should be evident that the hiring and firing policy of the optimising firm is discontinuous. In some periods the the optimal strategy of the firm will be to adjust hours of work. Under other demand conditions will be to fire or hire. More specifically, employment inaction will always be chosen when deviations of the expected marginal product of labour from the optimal level do not justify the costs of employment adjustment. In such situations, the firm prefers to adjust the actual hours of work, i.e. overtime work provides "flexibility at the margin". Adjustments to the workforce (hirings or firings) will only be observed when deviations in the expected marginal revenue product of labour from the optimal level are large enough to compensate for the hiring and firing costs. In other words, hiring and firing costs generate a corridor of inaction within which firms do not change their workforce. This region is identified by the upper, $Z_T$, and lower, $Z_F$, control barrier. The cyclical implications of this no action corridor are clear; firing costs increase employment in troughs and reduce employment in peaks. But it is unclear what the effects are on the average employment level. Bentolilia nad Bertola (1990) find in their model that firing costs actually

\footnote{$H_s$ is best interpreted as standard hours either set by the government or, more realistically, determined in the bargaining between firms and unions beyond which an overtime premium must be paid.}
increase average employment since the effect that they prevent firings dominates the effect from lower hiring. The question is whether or not this result is born out in our more general model allowing for overtime.

To determine the optimal labour demand policy of the firm one therefore needs to identify this no-action region, this involves calculating the optimal upper and lower control barriers as functions of the parameters of the model. There are no closed-form solutions to the model, but the real-options approach allows us to analyse changes in hiring and firing costs, changes in the overtime premium, and the implications of higher (lower) demand uncertainty (mean-preserving spreads) in numerical simulations.

B. The CES Case

Until now we have considered the case of a Cobb-Douglas technology with a unitary substitution elasticity. Below we analyse the role of the substitution elasticity. In the analysis of this issue we replace (1) with a more general three factor CES production function which has the form

\[ Y = \left[ \theta_K K^{-\mu} + \theta_H g(H)^{-\mu} + (1 - \theta_K - \theta_H)N^{-\mu} \right]^{\frac{1}{\mu}} \]

where \(-1 < \mu < \infty\) is the substitution parameter \(\mu \neq 0\), and \(0 < \theta_K, \theta_H < 1\) are the distribution parameters, \(H\) actual working hours, \(K\) is the capital stock and \(N\) is the number of employees. In (23) the elasticity of substitution is given by \(b = 1/(1+\mu)\). In equation (23) we have assumed constant returns to scale.\(^9\) The implicit demand function is again given by (2). Therefore, current profits are given by

\[ \Pi = Z \left[ \theta_K K^{-\mu} + \theta_H g(H)^{-\mu} + (1 - \theta_K - \theta_H)N^{-\mu} \right]^{\frac{1}{\mu}} \left[ w^\frac{1}{\mu} - w(H) + x \right] N \]

The firm chooses sequences \(\{N_t, H_t\}_{t=0}^\infty\) which solve the following optimisation problem:

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\(^8\) The overtime hours wages typically exceed the wages of standard hours although empirical evidence shows that this is not always the case, and sometimes there is even no compensation for overtime work. See, for instance, Trejo (1993) and Pannenberg and Wagner (2001).

\(^9\) It is apparent that the three-factor CES function is written in a "symmetrical" way for expositional convenience. In other words, there is no differential pattern of complementarity. Different degrees of complementarities are easiest to analyse in terms of a two-level CES function in which two factors (for example, \(H\) and \(N\)) are nested together in a subaggregate input \(X\) using one value of \(\mu\) and then \(X\) and \(K\) are entered into the main production function with a different and lower \(\mu\). Krusell et al. (2000) have recently used a similar specification for four inputs.
(25) \[ V = \max_{N, H} \int_0^\infty \left[ Z \left( \theta K K^{-\mu} + \theta H g(H)^{-\mu} + (1 - \theta K - \theta H) N^{-\mu} \right) - [w(H) + x] N \right] e^{-rs} ds \]

where \( r \) is the real rate of interest. It is assumed that workers never quit and the demand factor \( Z \) follows a geometric Brownian motion. Thus, the Bellman equation for the value \( V \) at time zero, in the continuation region is

(26) \[ rV = \max_{N, H} \left\{ Z \left( \theta K K^{-\mu} + \theta H g(H)^{-\mu} + (1 - \theta K - \theta H) N^{-\mu} \right) - [w(H) + x] N + \eta Z v + \frac{1}{2} \sigma^2 Z^2 v_{zz} \right\} \]

It is straightforward to see that the first-order condition for \( H \) is

(27) \[ \frac{\theta H}{\Psi} Z [g(H)]^{-\mu - 1} g'(H) \left( \theta K K^{-\mu} + \theta H g(H)^{-\mu} + (1 - \theta K - \theta H) N^{-\mu} \right) - [w(H) + x] N + \eta Z v + \frac{1}{2} \sigma^2 Z^2 v_{zz} = 0. \]

The shadow price of employees is represented by the equation

(28) \[ r_N V = \frac{1 - \theta_k - \theta_H}{\Psi} Z N^{-\mu - 1} \left[ \theta K K^{-\mu} + \theta H g(H)^{-\mu} + (1 - \theta K - \theta H) N^{-\mu} \right] - [w(H) + x] N + \eta Z v + \frac{1}{2} \sigma^2 Z^2 v_{zz} \]

where \( v = V_N \) is the value of employing the marginal worker. The particular solution is

(29) \[ v^p(Z) = -\frac{\frac{1 - \theta_k - \theta_H}{\Psi} Z N^{-\mu - 1} \left[ \theta K K^{-\mu} + \theta H g(H)^{-\mu} + (1 - \theta K - \theta H) N^{-\mu} \right] - [w(H) + x] N}{r - \eta} \]

The homogeneous solutions are the same as in equations (12) and (13). It remains to impose the value-matching and smooth-pasting conditions. When these conditions are imposed, we obtain the following system of equations for the hiring and firing thresholds in the CES case:

(30) \[ \frac{1 - \theta_k - \theta_H}{\Psi} Z T^* N^{-\mu - 1} \left[ \theta K K^{-\mu} + \theta H g(H)^{-\mu} + (1 - \theta) N^{-\mu} \right] - \frac{w(H) + x}{r} + \Lambda Z_T^{\beta_1} = T + \Lambda Z_T^{\beta_1} \]
Equation (30) – (33) are a straightforward generalisation of equation (15) – (18) for the Cobb-Douglas production function. In our intertemporal context, the firm’s forward-looking behaviour again anticipates future demand shocks, modifying employment policy for any particular period. The $g(H)$ and wage function are determined by equations (21) and (22) provided earlier. Again, the corresponding thresholds for hours ($H_T$ and $H_F$, respectively) can be calculated in a similar way. It is reassuring to know that the results of Section II can be replicated using a more realistic, but more cumbersome, apparatus. Having established this, we now turn to numerical simulations.

### III. Numerical Simulations

The preceding section has laid out the model economy. Having illustrated that the stochastic framework has important ramifications for the dynamic behaviour of employment, we proceed in this section to use the theoretical models derived above to carry out a number of comparative static analyses to shed light on the workings of the models and the economic forces at work. For this reason, the models are calibrated in order to match characteristics of the German economy. The use of consensus estimates ensures that the calibration is based on the best up-to-date knowledge in the literature. In this way, applied economic modelling is likely to increase the credibility of policy analysis.

The unit time length corresponds to one year. Our base parameters are $\sigma = 0.12$, $\eta = 0.0$, $K = 1$, $H_S = 1$, $x = 0.45$, $T = 0.1$, $F = 0.6$, $\Psi = 1.5$, $r = 0.08$, $\mu = 0.4825$, $\alpha = 0.3$, $\theta_K = 0.333$, $\theta_H = 0.333$, $w_S = 1$, $a = 1.5$, $\delta = \gamma = 0.9$. Where possible, parameter values are drawn from empirical labour studies. The firing and hiring parameters are consistent with those in Bertolila and Bertola (1990) for Germany. Their estimated firing costs for Germany are in the range $0.562 \leq F \leq 0.750$ and their hiring cost estimate (excluding on-the-job-training) for Germany is 0.066 of the average annual wage. Our specification ($T = 0.10$) is also broadly consistent with the recruiting and training cost of two months...
in Mortenson and Pissarides’ (1999) calibration. They suggest that this number is consistent with survey results reported in Hamermesh (1993). The substitution elasticity \( b = 1/(1+\mu) = 0.7 \) has been taken from Pissarides (1998). Hart (1984) documents that the share of quasi-fixed costs in labour costs is non-negligible. In line with Hart and Kawasaki (1988), we set \( x = 0.45 \). The overtime wage \( a = 1.5 \) is consistent with most German bargaining agreements and therefore a reasonable approximation to reality [see, for example, Bauer and Zimmermann (1999) and Trejo (1993)]. Finally, the price elasticity of demand parameter is set at \( \Psi = 1.50 \) as in Bovenberg et al. (1998).

We conduct various experiments to investigate the effects of uncertainty and/or policy variables upon employment and hours of work. First, we consider a policy which changes hiring and firing (layoff) costs.\(^{11}\) Despite the fact that liberalisation of labour markets has ranked highly in European policy debates, few effective changes to the stringent nature of the employment constraints facing European firms appear to have been implemented over the last decade. Moreover, in a number of European countries the general trend towards greater employment protection would actually appear to have continued.\(^{12}\) The results for alternative hiring and firing costs are given in Figure 1 and 2 below. Figure 1 presents the employment and hour thresholds for \( b = 1 \) (the Cobb-Douglas production function), while Figure 2 gives the optimal responses of a firm for \( b = 0.7 \) (the CES production function). In other words, Figure 1 - 2 report on the sensitivity of the calibration results to the specific production function used. The major results of the calibrations are that higher hiring and firing costs lead to an increase of the no action area, i.e. increasing hiring and/or firing increases the hiring threshold and decreases the firing threshold.\(^{13}\) The net impact upon employment turns out to be negative because the hiring thresholds are steeper, compared to the firing ones. The economic intuition for this result is the following. An increase in \( T \) has a direct effect on \( Z_T \) so that firms raise working hours \((H_Z)\) and then increase \( Z_T \). An increase in \( F \) leads to a reduction in firing options in \( Z_T \).

\(^{10}\) Note, however, that the goal of this paper is not to derive precise quantitative estimates of the impact of various labour market regulations, but rather to illustrate the qualitative predictions of a partial equilibrium model and to identify key features of the framework in determining the policy’s quantitative impact.

\(^{11}\) The numerical boundary value problem is solved with the method of Newton-Raphson for nonlinear systems. A description of the numerical programming technique is provided in Press et al. (2002).

\(^{12}\) One example is the recent French government’s remedy to introduce tighter labour laws. Assuming a company is not confronted by impossible circumstances or by irresistible technological change, it can announce redundancies only after all other means have been tried to preserve jobs. Moreover, it will have to negotiate with a work council authorised to offer other solutions and, if deadlock ensures, submit to the arbitration of a government-approved mediator. On the contrary, the Italian government is determined to alter Article 18 of the „worker statue“ dating from the 1970s. It requires employers to reinstate (not just compensate) workers whose dismissal is ruled unjust by the courts. Italian law goes further than any other in Europe in this respect and therefore is a deadweight on business.

\(^{13}\) The area of inaction exists because hiring and/or firing decisions are rarely a now-or-never decisions. In most cases, it is feasible to delay action and wait for new information, or at least begin with decisions that are limited in their scope and impact. Additionally it is important to note that the two optimal trigger functions interact, i.e. firing costs affect the value of the hiring trigger point. The reason is that higher firing costs make firms more cautious about hiring. Similarly, increasing hiring costs will increase the value of the firing trigger point.
and therefore firms turn to raise the working time \((H_Z)\) and hence increase \(Z_T\). This is an additional indirect effect which explains why the hiring thresholds are steeper, compared to the firing ones.

Another interesting feature is that \(H\) is smaller than \(H_S = 1\) for very large firing costs. In other words, when firing costs are very high, short-time work (a partial layout) turns out to be attractive for firms.

How does the CES-model, with its greater realism, modify the conclusion of the CD-case? Surprisingly, barely at all, as far as its qualitative employment properties are concerned. The widening of the no action corridor has important policy implications because it implies that demand can fluctuate much more without leading to changes in employment. This may explain why ceteris paribus unemployment is rather persistent in Europe and why firms in Europe are more reluctant to increase employment during business cycle upswings compared to the United States. Furthermore the modelling approach offers an explanation for the differences in employment across EU member countries with different employment protection legislation.\(^{14}\) Finally, reducing firing costs would assist job seekers in getting a foot into the labour market – especially those who would otherwise have little chance to do so: young, low-skilled, foreign and long-term unemployed. Reform which aim to deregulate the existing labour law, for example by allowing for more temporary contracts would serve as a bridge to permanent employment by offering firms a trial period in an uncertain environment.

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\(^{14}\) Nicoletti et al. (1999), pp. 40-50 have presented a database on indicators of employment protection legislation in the OECD countries as well as a methodology for aggregating these detailed indicators into summary indicators of the strictness of regulations.
Figure 1: The Employment ($Z_T$ and $Z_F$) and Hours ($H_T$ and $H_F$) Thresholds for Alternative Hiring and Firing Costs and $b = 1$ (Cobb-Douglas Production Function)
Figure 2: The Employment ($Z_T$ and $Z_F$) and Hours ($H_T$ and $H_F$) Thresholds for Alternative Hiring and Firing Costs and $b = 0.7$ (CES production function)
Second, we consider a policy which restricts the standard hours of all employees. In countries where unemployment (about 4 million in Germany) is seen as a national emergency, cutting working hours is becoming a popular solution. Germany’s most powerful trade union, IG Metall, is campaigning to cut the work week from 35 hours to 32 hours. How many jobs will be created by such a policy? The numerical results in Figure 3 for the CES production function and our baseline parameters indicate that a reduction of $H_S$ below 1 leads to a widening of the no-action-area. Again the effects are intuitively obvious. A cut in $H_S$ is qualitatively the same as an increase in fixed costs per worker, $x$. For given output, the marginal cost of an employee (the so-called extensive margin) rises but the marginal cost of an overtime hour (the intensive margin) remains constant, and so the firm substitutes away from employment towards hours. These results provide a warning about some potential, perhaps unforeseen, effects of a mandatory hours restriction. A reduction in standard hours not only leads to a decline in employment, but also results in an increase of overtime. In other words, the 35-hour week looks like a Trojan horse.

Figure 3: The Impact of a Reduction of Standard Hours ($H_S$) upon the Employment ($Z_T$ and $Z_H$) and Hours ($H_T$ and $H_F$) Thresholds

The fact that the firm varies labour utilisation because of high costs of labour adjustment, begs the question of workers willingness to incur the costs of short-term income variation and/or leisure variation. In other words, there are costs on both margins since deviations of marginal products from optimal levels can occur on both extensive and intensive labour margins. Our demand side approach may be argued to bias cost considerations towards the extensive margin. In effect the hiring and firing margins on the extensive margin may be matched by upper and lower thresholds of working time within the union’s utility function. For example, at the trough of the cycle, the firm may not be able to cut paid-for weekly hours as much as actual weekly hours because of the union’s reservation utility constraint. Adding this feature would require a general equilibrium approach.

This result is consistent with a survey among 1074 employees in western and eastern Germany. It shows that just 33 percent of all employees and 19 percent of full-time employees wish to work less than 35 hours per week. 53 percent of all employees complain that previous reduction of working hours have increased overtime work. See Schnabel (1997).
In Figure 4 we plot the firm’s optimal thresholds for different overtime premiums for the CES production function and our baseline parameters. The results indicate that a (an) decrease (increase) of the overtime premium leads to a widening (shrinking) of the no-action-area. For $a \geq 1.60$ firms do not ask workers to work overtime when demand conditions are relatively buoyant because it is not profitable any more. The reason why the width of the band depends upon $a$ is again fairly straightforward. When the overtime premium increases (decreases), the option value of hiring a new worker increases (decreases), the hiring cutoff decreases (increases), and firms hire more (less).\textsuperscript{17} Another interesting feature of the simulation results which emerges from Figure 4 is that $a$ has an asymmetric impact on the optimal employment and hours trigger points. More specifically, when the firms are confronted with lower overtime premiums, the hiring thresholds show a much more pronounced increase.

\textsuperscript{17} These numerical results imply that various recent flexible working deals between German firms and unions will have detrimental effects upon employment. These new „productivity deals“ typically require workers to forgo overtime pay and some bonuses, and to work extra hours when there is a big order-book, in return for paid time off when things are slack. Their basic pay is guaranteed at all times.
Next we compute thresholds for different values of the standard deviation, $\sigma \in [0.08, 0.18]$. The results are reported in Figure 5 below. Recall from section II that uncertainty enters into our model through changes in the evolution of demand strength, which is assumed to evolve according to a Brownian motion stochastic process.\[18\]

\[18\] In fact, the theory of real options defines in detail an economic model in which the variables directly affecting the information set of the firm are crucial. When this framework is applied to the employment issue,
Figure 5: The Impact of Uncertainty upon the Employment ($Z_T$ and $Z_H$) and Hours ($H_T$ and $H_F$) Thresholds

The results in Figure 5 indicate that the intertemporally optimising employer merely perceives there to be the possibility of a change in demand at some point in the future having an impact upon optimal employment. When firms perceive prevailing demand conditions to be transitory, in the sense that there are more frequent changes, then firms are more reluctant to hire or fire workers, i.e. a larger $\sigma$ will lead to a considerable widening of the no-action corridor. Conversely, smaller values of $\sigma$ results in a shrinking of the width of the corridor. In an increasingly transitory economic

the crucial variables responsible for the undertaking of hirings (firings) are likely to be those capable of sending a signal that reduces (increases) the level of uncertainty of the firm.
environment (increasing $\sigma$) the optimal employment strategy also implies a higher level of hours over the business cycle. In other words, firms opt for a wait and learn attitude in order to lower its degree of uncertainty.

In order to compare our results to those reported in Bentolila and Bertola (1990), we finally compare our results for the CES production function and our baseline parameters with the corresponding no overtime working case. The value-matching and smooth-pasting conditions for no overtime working are determined as follows. Since there are no overtime working and wage premium, we have

\begin{equation}
    g(H) = H_S^\gamma
\end{equation}

\begin{equation}
    w(H) = w_s H_S
\end{equation}

The value-matching conditions of equations (15) and (16) in the text become

\begin{equation}
    \frac{1-\theta_k-\theta_H}{\psi}N^{-\mu-1}\left[\theta_k K^{-\mu} + \theta_H H_s^{-\mu} + (1-\theta)N^{-\mu}\right]^{1-\mu}_{\psi} - \frac{w_s H_s + x}{r} = A_1 Z_T^\beta = T + A Z_T^\beta
\end{equation}

and

\begin{equation}
    -\left[\frac{1-\theta_k-\theta_H}{\psi}N^{-\mu-1}\left[\theta_k K^{-\mu} + \theta_H H_s^{-\mu} + (1-\theta)N^{-\mu}\right]^{1-\mu}_{\psi} - \frac{w_s H_s + x}{r}\right] + A Z_T^\beta = F + A Z_T^\beta
\end{equation}

and the smooth-pasting conditions become

\begin{equation}
    \frac{1-\theta_k-\theta_H}{\psi}N^{-\mu-1}\left[\theta_k K^{-\mu} + \theta_H H_s^{-\mu} + (1-\theta)N^{-\mu}\right]^{1-\mu}_{\psi} - \frac{w_s H_s + x}{r} + A_2 \beta Z_T^\beta = A_2 \beta Z_T^\beta
\end{equation}

and

\begin{equation}
    \frac{1-\theta_k-\theta_H}{\psi}N^{-\mu-1}\left[\theta_k K^{-\mu} + \theta_H H_s^{-\mu} + (1-\theta)N^{-\mu}\right]^{1-\mu}_{\psi} - \frac{w_s H_s + x}{r} + A_2 \beta Z_T^\beta = A_2 \beta Z_T^\beta
\end{equation}

Now we can simulate the model with and without overtime. The optimal trigger points under our assumption that firms can ask workers to work overtime with the hypothetical no-overtime case are given in Figure 6 below.
The graphs examine the direction and the magnitude of the bias which may arise though failure to control for endogenous variations in hours worked. The results indicate that demand fluctuates considerably more in our analysis than in the Bentolila and Bertola (1990) framework without warranting changes in employment because because firms can ask workers to work overtime when demand conditions are relatively buoyant. The results therefore indicate that particular care must be taken to ensure that misspecification of hours worked does not yield spurious estimates of the employment effects of firing and hiring costs and worksharing arrangements.\footnote{The comparison of both regimes indicates that higher $F$ and $T$ also leads to a more pronounced increase of the width of the no action corridor.}

IV. Summary and Discussion

In this paper it has been proven that the theory of real options when applied to the case of employment and working time determination can be a fruitful extension of the traditional deterministic framework since it is able to combine consistently the existing interactions between irreversibility, uncertainty and the choice of timing, all peculiar characteristics of an employment decision. It can therefore be concluded that new theories of employment have to rest upon an approach, a sort of new agenda for modelling employment and hours of work stressing dynamic issues such as uncertainty, market volatility and the expectations and beliefs of firms.\footnote{One should, however, be aware that, notwithstanding this refinement, the theoretical framework above cannot be taken as a general tool of interpretation of the complex and multifaceted phenomenon of}
have not formally tested the forward-looking model under uncertainty, we have sketched how its predictions might be consistent with aggregate data. One qualitative result was that any reduction in hiring and firing costs acts as a signal able to lower the uncertainty of the firms, increasing the opportunity cost of waiting and therefore fostering the undertaking of hiring and/or firing decisions. Another interesting result is that reductions of weekly standard hours have but small employment effects. Nevertheless, German trade unions still propose such a strategy for reducing unemployment. Our results suggest that in order to overcome Germany’s massive employment problems, defensive reductions of working hours should be replaced by offensive strategies for increasing labour market flexibility and stimulating economic growth.

Before finishing we note a few caveats. First, the paper has focused on the effects of an hours restriction on labour demand decisions, and abstracts from numerous potentially important labour supply considerations. Second, we have not introduced heterogeneity into the analysis, i.e. we have not considered a skill mismatch between the skill characteristics of the employed and the unemployed. Third, we have not considered fixed-term contracts which give employers the opportunity to dismiss a worker with low firing costs when the contract expires. Finally, general equilibrium considerations are absent. We leave these extensions for future research.
References:


