Cumulative Causation, Capital Mobility and the Welfare State

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and

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Abstract

Within a small open economy with vertical linkages, welfare state policies trigger a virtuous circle of cumulative causation that will lead to higher levels of economic activity by improving the exploitation of potential aggregate scale economies. Capital mobility is typically found to reinforce this mechanism and the use of capital taxation to finance redistribution policies is not found to alter these conclusions. These results, consistent with the evidence that welfare states and tax burden have not significantly reduced in OECD countries, challenge the conventional wisdom that globalisation undermines governments’ ability to pursue income redistribution.

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1. INTRODUCTION
This paper studies how welfare state policies – in the form of unemployment benefits – affect economic performance in an open economy in the presence of international capital mobility and aggregate increasing returns to scale.

Industrialisation has been accompanied worldwide by an increase in the use of fiscal redistribution. However, during the 1980s and 1990s, the role of government in the economy has come under serious pressure. Welfare state policies in particular have been attacked for their distortionary effects on the economy and are often deemed responsible for high unemployment and low growth rates. These concerns have been heightened by economic globalisation. On the one hand, welfare programmes are seen as having an adverse effect on firms’ costs, thus hindering their ability to successfully compete in global markets (e.g. Alesina and Perotti, 1997). On the other hand, economic integration is perceived as reducing national governments’ ability to independently shape their economic and social policies. The increasing degree of capital mobility currently characterising the world economy lies at the core of this argument. The credible threat of exit of capital and firms is seen as reducing national control over both volume and structure of the tax revenue by leading to a shrinking of the tax base and to pressures to shift the burden of taxation on to less mobile factors such as labour, as governments compete with each other to attract/retain international investment.

Consistently with the normative implications of this conventional wisdom, attempts to roll back the welfare state in western democracies in the last twenty years – which have not been limited to centre-right governments – have often been portrayed as an inevitable answer to the pressures of an increasingly integrated world economy.

Two stylised facts, however, are somewhat at odds with this conventional wisdom. First, overall burdens of taxation in advanced industrial economies between the mid-1960s and the mid-1990s do not appear to have significantly reduced as a result of market integration and, although labour taxes as a proportion of government revenue have grown faster than capital taxation, the average effective tax rates on capital have increased in OECD countries (OECD 1996, Baldwin R. and Krugman, 2000; Garrett and Mitchell, 2001; Swank, 2002). Second, no strong evidence can be found that the increased extent of goods and capital market integration has contributed systematically to the retrenchment of developed welfare states (see for instance, Rodrik, 1997; Garrett, 1998; Swank, 2002). Reforms have generally been limited to a restructuring of expenditure and whilst some areas of social protection have modestly declined, others have experienced slow growth or stability. This can partly be ascribed to the resistance
attempts to reforming the welfare state have met in western societies. As some authors – e.g. Garrett (1998) and Rodrik (1997, 1998) – have pointed out, increasing integration of goods and capital markets might heighten the needs for social insurance and income redistribution in response to internationally generated risk and economic dislocations.

More generally, however, the effects of the globalisation of markets on countries’ economic performance and on their ability to retain control on domestic policies will be shaped by a complex set of circumstances, many of which will be country specific. Political scientists (e.g. Garrett, 1998; Swank, 2002) argue convincingly that the extent to which the economic and political pressures stemming from globalisation are translated into welfare state retrenchment depends on the institutional features of both the socio-political representation system (e.g. type of electoral and interest representation) and the welfare state (e.g. its degree of universalism).

In this paper, we identify the production structure – and in particular the existence of aggregate scale economies – as an additional factor that contributes to shaping the relationship between welfare state policies and economic globalisation. Contrary to what the conventional wisdom suggests, we contend that economic integration does not inevitably require a rolling back of the system of social protection via a reduction of the revenue-raising capacity of governments and a deterioration of firms’ competitiveness. Instead, we show how welfare state policies may complement rather than be in conflict with international openness in improving economic performance, thus enhancing the ability of governments to sustain programmes of public expenditure.

The central idea of our argument is that social security programmes may lead to higher levels of economic efficiency by improving the exploitation of potential aggregate economies of scale. This channel is particularly relevant in mature industrial countries due to the high complexity of their economic systems that stems from unprecedented depths of the division of labour and results in production externalities whose effects on the economy may not be easily predicted. We argue that the acknowledgment of these externalities is essential for any meaningful debate about the sustainability of welfare state programmes.

The division of labour within the economy is modelled by means of an input-output production structure whereby final good sectors use intermediate inputs produced by a monopolistically competitive upstream industry. The vertical linkages between sectors result in aggregate increasing returns to scale to the range of intermediate varieties.

\footnote{In Acemoglu and Shimer (2000), unemployment insurance improves the allocation of resources by enabling workers to pursue riskier and more productive options.}
By allowing for both final goods and capital to be internationally mobile, we contribute to assessing the view that factor mobility poses an additional threat to the sustainability of the welfare state. International capital flows, which are clearly affected by redistribution policies and their financing, represent an additional channel through which economic policy can influence the depth of the division of labour within the economy and the country’s pattern of international specialisation. The nature of the interaction between capital mobility and fiscal policy in the presence of increasing returns, however, is not obvious and may give rise to counterintuitive results. We show that, by enhancing the exploitation of aggregate scale economies, a more generous welfare state increases overall welfare regardless of the tax instrument used to offset the shock. In particular, the effects on aggregate income can be positive even when the increase in the welfare bill is financed through an increase in capital taxation that leads to a capital outflow. We also find that a higher capital tax rate does not necessarily lead to an outflow of capital. These findings, which clearly challenge the view that capital mobility undermines governments’ ability to pursue income redistribution, are consistent with and help explain the two stylised facts mentioned above.

The rest of the paper is organised as follows. Section 2 outlines the model, Section 3 describes the general equilibrium and Section 4 carries out the policy analysis. Section 5 provides an extension of the model and Section 6 draws some conclusions.

2. THE MODEL
2.1. The theoretical framework
Since international economic integration is perceived as reducing the economic size of countries and their monopoly power in world markets, we shall analyse the effects of welfare state policies on economic performance within a small open economy framework and assume free international trade and capital mobility.

Given that welfare states are primarily a defining feature of advanced industrial countries, we shall assume that – as is typical of these countries – our small open economy has (1) a government that pursues redistributive policies financed through distortionary taxation, (2) unionised labour markets, and (3) a production structure characterised by intersectoral linkages that give rise to aggregate economies of scale.

It is now widely accepted that a typical implication of industrial development is the increasing complexity and ‘indirectness’ of production processes, with final goods sectors relying more and more on highly specialised intermediate inputs. We therefore assume an input-
output structure\(^2\) with one upstream monopolistically competitive industry and two downstream perfectly competitive sectors producing two homogenous final goods (\(Y_1\) and \(Y_2\)). The output of the upstream industry (\(X\)) comes in a continuum of horizontally differentiated varieties that can be thought of as consisting of highly specialised producer services and other intangible inputs such as knowledge. The larger the number of intermediates, the higher is the degree of specialisation in production and the resulting aggregate efficiency. Thus, to the extent that government policies influence market structure and the availability of intermediates, they will affect aggregate productivity, the economy’s trade performance and the direction of international capital flows.

2.2. Consumers
The representative individual, who is endowed with one unit of labour supplied inelastically, has utility function

\[
U = \frac{Y_1^\mu Y_2^{1-\mu}}{(1-\mu)^{1-\mu} \mu^{-\mu}} + (1-\xi)\bar{V}, \quad (1)
\]

and is employed if \(\xi = 1\) and unemployed if \(\xi = 0\). In (1), \(Y_h\) (\(h=1,2\)) are the quantities consumed of final goods and \(\bar{V}\) is the utility of leisure. Constrained optimisation of (1) yields the demand functions

\[
Y_1^d = \mu \frac{M}{P_1},
\]

\[
Y_2^d = (1-\mu) \frac{M}{P_2}, \quad (2)
\]

where \(P_h\) (\(h=1,2\)) are the prices of the two goods and \(M\) is nominal disposable income (to be defined later).

2.3. Producers
There are three primary inputs in the economy that we call labour (\(L\)), land (\(Z\)), and capital (\(K\)) whose rates of return are respectively denoted by \(w\), \(q\), and \(r\). Whilst labour (which is specific to

\(^2\) For similar models see for example Rodrik (1996) and Rodriguez-Clare (1996). Existing empirical evidence reveals important inter-industry connections leading to external returns to scale in manufacturing (e.g. Caballero and Lyons, 1992; Bartelsman, Caballero and Lyons, 1994). The theoretical importance of vertical linkages as a source of economy-wide increasing returns to scale has been widely acknowledged, e.g. Eithier (1982), Matzuyama (1995), Venables (1996).
the intermediate sector) and land are internationally immobile, capital mobility is initially allowed for (the implications of relaxing this assumption are discussed in Section 5). We shall treat $K$ as physical capital but, as we shall argue, the results of the analysis would not be qualitatively affected by assuming $K$ to be human capital instead.

The downstream industries produce two homogenous consumer goods $Y_1$ and $Y_2$ which are freely traded in world markets and which we shall label ‘high-tech’ and ‘low-tech’ respectively. Labour is not directly required as a primary factor in the production of either good. Instead, both commodities are obtained using $Z, K$ and a basket $X$ of intermediate inputs. For a given set of intermediates, both final goods are produced with a constant returns to scale Cobb-Douglas technology $Y_h = (\alpha_h^\lambda \beta_h^\lambda \lambda_h^\lambda) K_h^\lambda Z^\lambda X_h^\lambda (h=1,2)$, where $0 < \alpha_h < 1, 0 < \beta_h < 1$ and $\lambda_h = 1 - \alpha_h - \beta_h$. It is plausible to assume that the high-tech good is relatively more intensive in the intermediates and at least as intensive in capital as the low-tech good, which is instead relatively more intensive in factor $Z$.

Hence, $\frac{\beta_2}{\beta_1} < \frac{\alpha_1}{\alpha_2} < \frac{\lambda_1}{\lambda_2}$.

The intermediate input, assumed to be non-traded, consists of a mass $N$ of horizontally differentiated varieties that are assembled into a CES composite input $X = \left( \int_{i \in N} x_i \sigma^{-1} \right)^{\sigma^{-1}}$, where $x_i$ is the quantity of a typical variety $i$ and $\sigma$ is the elasticity of substitution between varieties. We assume $\sigma > 1$, which means that no single variety is an essential input per se. This CES technology implies that there are increasing returns to the range of available varieties, since the productivity of the intermediate basket rises in $N$. Also note that the increase in the average productivity of the intermediates stemming from a given increase in $N$ will be higher the smaller is $\sigma$.

The intermediate varieties are produced by an endogenously determined (via free-entry and exit) mass of identical firms. The existence of a fixed production cost, by giving rise to internal increasing returns to scale and to an incentive to specialisation, leads to a one-to-one correspondence between the mass of firms and that of available varieties. Labour is the only

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3 In addition to its technological relevance, this assumption helps limit the combination of possible cases when determining the effect of a policy shock. The results would not be qualitatively affected if we assumed capital to be used most intensively in the low-tech sector.

4 The non-tradability of intermediates is commonly assumed in the literature to capture the importance of geographical proximity of the intermediate sector to final good industries (e.g. Rodriguez-Clare, 1996, and Rodrik, 1996). Note, however, that in the presence of inter-sectoral linkages, this assumption does not imply that upstream sector producers are shielded from international competition.
factor of production, used as both fixed and variable input. The labour requirement of a typical firm \(i\) is

\[ l_i = \delta x_i + \gamma, \quad (3) \]

where \(\delta > 0\) and \(\gamma > 0\) are the marginal and fixed input coefficients. The firm’s profit is

\[ \pi_i = p_i x_i - w_i l_i, \quad (4) \]

where \(x_i\) and \(p_i\) are the firm’s output and price and \(w_i\) is the wage rate it pays its workforce.

The price index for the intermediate varieties, dual to the CES basket defined above, is

\[ P_s = \left( \int_{i=N}^{1} p_i^{1-\sigma} \, dp_i \right)^{\frac{1}{1-\sigma}}. \quad (5) \]

Given the downstream sectors’ production technology, the minimum total cost of producing \(Y_h\) is

\[ C_h = Y_h \left( r^{\alpha_h} q^{\beta_h} P_s^{\lambda_h} \right), \quad h=1,2. \]

Since these two industries are perfectly competitive, their production level is determined by the equality between price and average cost:

\[ P_h = r^{\alpha_h} q^{\beta_h} P_s^{\lambda_h}. \quad (7) \]

The small open economy and free trade assumptions imply that final good prices are determined in world markets.

The constant returns to scale technology and the perfect competition assumption imply that input demands in the two final good industries’ are

\[ \begin{align*}
X^{d}_{h} &= \lambda_h Y^d_h \frac{P_h}{P_s}, \\
Z^{d}_{h} &= \beta_h Y^d_h \frac{P_h}{q}, \quad h=1,2 \\
K^{d}_{h} &= \alpha_h Y^d_h \frac{P_h}{r}
\end{align*} \quad (8) \]

Finally, a system of demand equations for the varieties of the intermediate good by downstream producers can be obtained, given (5), by applying Sheppard’s Lemma to (6):

\[ x_i = \left( X^d_1 + X^d_2 \left( \frac{P_i}{P_s} \right)^{\sigma} \right). \quad (9) \]
2.4. Factor markets

In the markets for land and capital, which are assumed to be perfectly competitive, factor prices \(q\) and \(r\) adjust to satisfy the market clearing resource constraints:

\[
Z = Z_1^d + Z_2^d, \tag{10}
\]

\[
K + K^* = K_1^d + K_2^d, \tag{11}
\]

where \(Z\) and \(K\) are the economy’s endowments of these factors. With capital mobility, the stock of available capital can differ from the country’s endowment by an amount \(K^*\) that denotes the capital inflow/outflow.

The labour market in the intermediate sector is unionised, with unions having monopoly over wages and firms determining employment levels\(^5\). We assume there to be \(J\) symmetric unions, with a large (small) \(J\) indicating a large (small) number of small (large) unions. A typical union \(j\) embraces the workers of and sets the wage for a mass of firms \(N_j = N/J\) and will therefore have a mass of members \(L_j = L / J\), where \(L\) is the total labour endowment of the country. Hence, \(w_{wN_j} = w_j\). Note, that alternatively and equivalently, one could think of the union structure as consisting of a mass \(N\) of firm specific unions grouped in a discrete number \(J\) of clusters of unions, with unions within each group setting the cooperative wage for the \(N/J\) firms they cover and behaving non-cooperatively with respect to the other clusters.

With unionisation, labour income taxes will have distortionary effects on the economy even if the individual labour supply is perfectly inelastic, since unions will transfer part of the burden of taxation on to employers via higher wages. The degree of distortion, however, is known to depend on the institutional settings of the labour market\(^6\). This is because rent-seeking unions face a trade-off between the exploitation of firms’ monopoly power (which works in the direction of higher wages), and the acknowledgement of the negative externalities of higher wages on employment and real disposable income (which generates incentives for wage restraint). As their size and/or the degree of coordination of their actions increase, the extent to which unions internalise the effects of their wage decisions on the macroeconomic constraints rises. To capture the fact that unions’ behaviour changes as they become larger and/or more coordinated, we follow Alesina and Perotti (1997) and consider two discrete cases, which we shall refer to as the non-internalisation and internalisation. In the first case, the number of

\(^5\) The labour market is modelled as in Alesina and Perotti (1997).

\(^6\) See, for instance, Calmfors and Driffill (1988); Summer, Gruber and Vergara (1993) and Rama (1994).
unions is sufficiently large and the typical union \( j \) is sufficiently small for it not to internalise the consequences of its actions for the government budget constraint. In the second, the number of unions is sufficiently small and the typical union \( j \) is sufficiently large for it to internalise the consequences of its actions for the government budget constraint. In both cases, unions will take into account the effects of their wage setting decision on the intermediate industry price index.

Unionisation implies that involuntary unemployment persists in equilibrium and that each union will have some unemployed members\(^7\). The objective function of a typical union \( j \) can be obtained from (1) and is given by the expected utility of its typical member,

\[
V_j = \frac{L_j (1 - \tau) w_j}{P} + \frac{\bar{L}_j - L_j}{P} b + \frac{\bar{L}_j - L_j}{P} \bar{\nu},
\]

where \( L_j \) is the union’s employed membership, \( P = P_1 P_2^{1-\mu} \) is the consumer price index, \( \tau \) and \( b \) are respectively the labour income tax and the unemployment benefit rates. The latter, which we assume not to be taxed, is a net transfer. As we shall explain later, the union will choose \( w_j \) to maximise (12) subject to the relevant constraints.

2.5. Government

The government provides welfare protection in the form of unemployment benefits, financed through the taxation of the primary factors’ income. We use the source principle as a tax rule, so that income generated by the inflow of capital is taxed before it is repatriated. Noting that \( \sum_{j=1}^J L_j \geq L = \sum_{j=1}^J L_j \), the government budget constraint is therefore given by:

\[
b(\bar{L} - L) = \sum_{j=1}^J b(\bar{L}_j - L_j) = \sum_{j=1}^J \tau w_j L_j + \rho r(\bar{K} + K^*) + \phi q \bar{Z},
\]

where \( \phi \) and \( \rho \) are respectively the land and capital income tax rates.

Finally, aggregate income \((M)\) is determined by total returns to primary factors and transfers between the public and private sectors\(^8\):

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\(^7\) We follow the literature in assuming that unemployed workers from other unions cannot be employed in a given union’s sector before the latter’s unemployed members are hired.

\(^8\) Equations (13) and (14) reflect the assumption that \( K \) is physical capital. Were \( K \) to represent human capital (e.g. skilled labour), the return to the factor would be spent where it is located. It is straightforward to show that, given our small open economy assumption, treating \( K \) as human capital (and assuming that – perhaps due to its international mobility – it is not unionised) would not have any qualitative effects on the results (the only effect being a shift in income brought about by the non-repatriation of the returns to the factor).
\[ M = \sum_{j=1}^{J} \left[ (1 - \tau) w_j L_j + b (\bar{L}_j - L_j) \right] + (1 - \rho) r K + (1 - \phi) q Z. \]  \hspace{1cm} (14)

2.6. Foreign sector

The balance of payments equation is

\[ P_1 (Y_1^s - Y_1^d) + P_2 (Y_2^s - Y_2^d) - (1 - \rho) r K^* = 0. \]

Assuming perfect substitutability and full mobility of capital, arbitrage in the international capital market ensures that the interest parity condition holds

\[ (1 - \rho) r = r^*, \]  \hspace{1cm} (15)

where the left-hand-side is the net of tax domestic interest rate and the right-hand-side is the (exogenous and net) foreign rate of interest.

3. GENERAL EQUILIBRIUM

Given equations (3), (4) and (9), and taking as given the wage set by the unions, the optimal price rule for a typical firm \( i \) covered by union \( j \) will be given by

\[ p_{wN_i} = p_j = \left( \frac{\sigma}{\sigma - 1} \right) \delta w_j. \]

Adopting for simplicity the normalisation \( \frac{\sigma - 1}{\sigma} = \delta \), firms’ optimal price setting rule can be written as \( p_j = w_j \).

Equilibrium wages are determined by the monopoly unions. In the non-internalisation case the typical union \( j \) maximises (12) subject to the labour demand function it faces, given by

\[ L_j^d = \int_{i \in N_j} l_i d i, \]

taking account of (3), (5) and firms’ mark-up rule \( p_j = w_j \). In the internalisation case, in addition to the above, the union also takes into account the government budget constraint in (13).

Note that in maximising their objective function unions take the mass of firms and the government policy variables as given, which is equivalent to assuming that entry into the industry and the government choice of policy instruments occur prior to unions’ setting of wages.

Denoting the ‘non-internalisation’ and the ‘internalisation’ of the government budget constraint with the superscript ‘NI’ and ‘I’ respectively, the corresponding wage setting equations that result from union \( j \)’s optimisation in the two cases will be
\[ w^N_j = \frac{b + P\tilde{V}}{(1 - \tau)\left(1 - \frac{1}{\varepsilon_j}\right)} \]  

(16a)

\[ w^I_j = \frac{P\tilde{V}}{\left(1 - \frac{1}{\varepsilon_j}\right)} \]  

(16b)

The numerators in (16a) and (16b) give the respective reservation wage and \[ \varepsilon_j = -\frac{dL^d_j}{dw_j} w_j \]  is the wage elasticity of labour demand facing a typical union and provides an inverse measure of its monopoly power. It can be shown that \[ \varepsilon_j = \varepsilon = \sigma - (\sigma - 1) / J \]  (see the Appendix A.1) from which follows that under each unionisation regime, \( w_j = w \) \( \forall j \). The mark-up over the reservation wage, \( \varepsilon / (\varepsilon - 1) \), is negatively related to \( \varepsilon \), and therefore falls in \( J \). Hence, ceteris paribus, as unions become larger, their monopoly power and their wage demands increase.

In (16a) the wage rate is positively related to both the tax and unemployment benefit rates. A ceteris paribus increase in \( \tau \), by reducing the after tax wage, induces the unions to bid up the nominal wage. A higher \( b \) – by reducing the utility difference between being employed and unemployed – allows the unions to increase their wage demands. However, when unions are large enough to internalise the effects of their actions on the government budget constraint the wage is independent of both income tax and unemployment benefit rates. It follows that, for a given \( J, w^I < w^N \) always holds. More generally, if – as is plausible – the number (size) of unions is smaller (larger) in the internalisation than in the no-internalisation case (i.e. \( J < J^N \)), then \( w^I < w^N \) if \( \tau \) and \( b \) are sufficiently large. Note that letting \( J \to \infty \) and \( \varepsilon \to \sigma \), the wage in equation (16a) – which becomes independent of \( J \) – tends to its lower bound \( w \). It is straightforward to show that this limiting case corresponds to the very decentralised situation in which unions are so small that they even disregard the effects of their actions on the industry price index. It then follows that our model conforms with the humped-shaped relationship between the wage rate and the degree of wage setting centralisation found in the literature, because \( w < w^N \) and the latter, which rises with unions’ size, is typically larger than \( w^I \).  

9 Note that unions’ first order conditions are derived before imposing symmetry between unions’ wages.

10 The humped-shaped relationship between the degree of domestic coordination and wage levels is shown in the literature to be ‘flattened’ by foreign competition, because the more open is the economy the lower will be the impact on consumer price index (CPI) of domestic wage changes, given that the latter will be more heavily dependent on foreign prices. In the limit, as in the small open economy with free trade discussed in this model, the CPI, \( P = P_1^\mu P_2^{1-\mu} \), is fully exogenous. As a result, the relationship between wage and union power and
Note that the symmetry of wages in equilibrium (i.e. \( w_j = w \)) implies that \( p_j = p \ \forall j \).

Hence the optimal price rule will be for all firms to set

\[
p = w. \tag{17}
\]

In the free-entry equilibrium, each firm will break even. Substituting from (3) into (4), setting the resulting equation equal to zero and using (17) and the normalisation \( \frac{\sigma - 1}{\sigma} = \delta \), we obtain the equilibrium output scale of a typical firm in the intermediate good industry, which, in the symmetric equilibrium, will be

\[
x = \sigma \gamma, \tag{18}
\]

which is constant and does not depend on market size. This is due to the constant elasticity of substitution assumption and the lack of strategic interaction between firms, whereby the extent to which each firm exploits internal increasing returns to scale depends only on the elasticity of substitution between varieties. Hence, changes in the size of the market do not affect the mark-up and the optimal output scale of firms but only work through changes in the product range \( N \).

The general equilibrium of the model described above consists of the following equations\(^{11}\) where (15), (16a) and (16b) are repeated for convenience:

\[
(1 - \rho) \hat{r} = \hat{r}^*, \tag{15}
\]

either

\[
w^{NI} = \frac{b + P\hat{\nu}}{(1 - \tau)(1 - \frac{1}{\varepsilon})}, \tag{16a}
\]

or

\[
w' = \frac{P\hat{\nu}}{(1 - \frac{1}{\varepsilon})}, \tag{16b}
\]

\[
P_x = \left(p_1, p_2^{*} \right)^{\frac{1}{\gamma}} \theta, \tag{19}
\]

\[
q = \left(p_1^{*}, p_2^{*} \right)^{\frac{1}{\gamma}} r^{\theta}, \tag{20}
\]

\[
P_x = N^{\frac{1}{\gamma}, \theta} w, \tag{21}
\]

degree of internalisation could even be monotonic. To see this recall that \( w^{NI} < w' \) is possible for certain values of \( \tau, b, \hat{J}^{NI} \) and \( \hat{J} \).

\(^{11}\) By Walras’ law, the balance of payment equation can be obtained from (13), (14) and the other market equilibrium conditions.
\[ L = y\sigma N , \]  
\[ P_X X = wL , \]  
\[ r (K + K^*) = \theta \gamma qZ - \theta _t P_X X , \]  
\[ b (\bar{L} - L) = \alpha w L + \rho r (K + K^*) + \phi qZ . \]

Equations (19) and (20) are obtained by rearranging (7), where \( \theta _{+} = (\alpha _2 \beta _1 - \alpha _1 \beta _2) / \Delta < 0, \) \( \theta _{+} = (\alpha _2 \lambda _1 - \alpha _1 \lambda _2) / \Delta > 0, \) and \( \Delta = \beta _2 \lambda _1 - \beta _1 \lambda _2 > 0; \) (21) is obtained from (5) and (17) in the symmetric equilibrium; (22) is the total labour demand obtained by aggregating (3) and (18); (23) is the zero profit condition obtained by setting the aggregate version of (4) equal to zero; (24) is the overall resource constraint obtained by eliminating \( Y^1 \) and \( Y^2 \) from (8), (10) and (11) (see Appendix A.2); and (25) is the government budget constraint in (13).

For given parameters \( (\alpha _1, \alpha _2, \beta _1, \beta _2, \lambda _1, \lambda _2, \gamma, \sigma) \) and exogenous variables \( (Z, K, L, P_1, P_2, r^*, b, \bar{V}, \varepsilon, J) \), the above equations can be solved to determine the endogenous variables \( q, r, w, P_X, K^*, \) and one of the tax rates \( \tau, \rho, \) or \( \phi \) which the government will allow to vary to balance its budget. For simplicity, let \( \phi \) be the endogenous tax rate. It is then easy to show that the model can be solved recursively: \( w \) and \( r \) are determined by (16) and (15), and (19)-(25) then determine \( q, P_X, N, L, X, K^* \) and \( \phi \) in that order.

Before analysing the consequences of a move to a more generous welfare system, it is useful to highlight the main characteristics of the working of this economy by considering the effects of an exogenous increase in the wage rate (e.g. due to a fall in \( J \)) and assuming for simplicity that \( \phi \) is the endogenous tax rate. Given the small open economy assumption, for a given \( \rho, \) the interest parity condition and the exogeneity of final good prices imply that \( r, q, \) and \( P_X \) are ultimately determined regardless of the rest of the model — see equations (15), (19), (20). It follows from equations (21) and (22) that ceteris paribus increases in the wage rate are associated with a larger mass of firms and a higher aggregate employment, as long as \( \sigma \) exceeds unity and is finite.

The positive relationship between the wage rate and the number of firms and employment is not intuitively obvious and critically rests on the existence of increasing returns to the range of available intermediates which implies that the equilibrium mass of varieties (and firms) may be
sub-optimal\textsuperscript{12}. In these circumstances, a higher wage may contribute to alleviating market inefficiencies.

For a given mass of firms $N$, the immediate impact of an increase in $w$ will be to raise the marginal production cost in the upstream sector. Firms in the sector will react by adjusting prices upwards. As a result, the industry price index $P_x$ will initially go up, leading – in both final good sectors – to a substitution away from $X$ and towards $K$ and $Z$, hence exerting an upward pressure on these factors’ prices and leading to an incipient inflow of capital. On the whole, this first effect will entail lower profits for intermediate producers and will clearly work towards a reduction of the mass of firms in the industry.

However, the higher wage will also crucially imply a higher disposable income thus triggering an aggregate demand effect that is essentially Keynesian in nature. By stimulating consumption of final goods, the higher wages will lead to a higher demand for all factors of production. In particular, the higher demand for $X$ will foster entry into the upstream sector and contribute to counteract the adverse effects of the increase in firms’ marginal cost on the mass of available varieties. The further increase in the demand for $K$ (and $Z$) – by strengthening the upward pressure on the domestic interest rate – will reinforce the capital inflow until the interest parity condition is restored. The larger capital stock will in turn increase the marginal product of the other factors, thus boosting the demand for $X$.

The overall effect of these forces will be a net increase in the demand for the upstream good, a larger mass of firms in the industry and a deeper division of labour within the economy. As a result of the expansion in the range of intermediates, average production costs will decline in both downstream industries – but particularly so in the production of $Y_1$ which is relatively intensive in the intermediate input. Resources will shift from the low-tech to the high-tech sector, but the contraction of the former will release less intermediates (and less capital, if the high-tech good is also relatively intensive in capital – i.e. if $\alpha_1/\alpha_2 > 1$) than the high-tech sector requires at the given factor prices. The excess demand for $X$ and $K$ implies further entry of firms in the upstream industry and of capital into the country. In sum, the original wage shock will result in a virtuous circular causation process of rising demand for intermediates, entry of new firms into the upstream sector, and increased specialisation in the high-tech good that will amount to an increase in the level of economic activity, i.e. higher employment in the upstream sector and higher output of intermediates and final goods.

\textsuperscript{12} See for instance Devereux, Head and Lapham (2000).
Since $w_{NI} > w^J$ typically holds, the positive relationship between $w$ and $N$ implies that the mass of firms and aggregate employment will be larger when unions do not internalise the government budget constraint.

4. THE EFFECTS OF WELFARE POLICIES

This section analyses the policy multipliers resulting from an increase in the rate of unemployment benefit, when different tax instruments are used to offset the implications of the policy shock for the government budget constraint. In all cases we consider the two types of unionisation discussed above.

It is analytically convenient to start from an initial equilibrium in which $K^* = Y_1^* = Y_2^* = 0$, where $Y_1^*$ and $Y_2^*$ are the excess supply of the high-tech and of the low-tech good respectively (see Appendix A3). It is easy to show that these initial conditions do not qualitatively affect the results. All multipliers have been derived analytically (see Appendix A5-A6 for details). For expositional simplicity in what follows we shall concentrate on the main qualitative effects of an increase in unemployment benefits financed via changes in the taxation of one of the primary factors of production. In the summary tables below, the first column gives the tax instrument used ($\eta = \tau, \phi, \rho$).

4.1. Unions do not internalise the government budget constraint

Table 1 summarises the results for the case in which unions do not internalise the government budget constraint.

<table>
<thead>
<tr>
<th>Tax rate used (\eta)</th>
<th>$\frac{dw}{db}$</th>
<th>$\frac{dN}{db}$</th>
<th>$\frac{dX}{db}$</th>
<th>$\frac{dL}{db}$</th>
<th>$\frac{dK^*}{db}$</th>
<th>$\frac{dM}{db}$</th>
<th>$\frac{dY_1^*}{db}$</th>
<th>$\frac{dq}{db}$</th>
<th>$\frac{dP_x}{db}$</th>
<th>$\frac{dr}{db}$</th>
<th>$\frac{dWB}{db}$</th>
<th>$\frac{d\eta}{db}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau$</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>+</td>
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<tr>
<td>$\phi$</td>
<td>+</td>
<td>+</td>
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<td>+</td>
<td>+</td>
<td>+</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>+</td>
<td>$^{(4)}$</td>
</tr>
<tr>
<td>$\rho$</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>$^{(6)}$</td>
<td>+</td>
<td>$^{(3)}$</td>
<td>+</td>
<td>$^{(3)}$</td>
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(1) See Appendix A5 and A6 for derivations and analytical expressions. WB is the total unemployment benefit bill and $Y_1^*$ is the excess supply of good 1.
(2) The necessary condition for a positive sign is that the propensity to consume $Y_1$ is sufficiently small. See Appendix A6.1.1 and A6.1.2 for details.
(3) A positive sign is obtained if $\sigma$ and (or) $\tau$ are sufficiently small. See Appendix A6.1.1 for details.
(4) A positive sign is obtained for sufficiently small values of $\sigma$. See Appendix A6.1.2 for details.
(5) These signs are obtained by numerical simulations with plausible calibrations (see Appendix A6 for initial values and plots of the multipliers).
(6) This multiplier can be negative for sufficiently small values of $\sigma$. 

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When the shock is offset using the tax rate on one of the immobile factors, a rise in the rate of unemployment benefit will unambiguously increase the equilibrium wage, the range of intermediate varieties (i.e. the depth of the division of labour), and the country’s income, and will typically lead to a higher degree of specialisation in the high-tech good. The policy will also unambiguously result in a capital inflow.

To explain these results recall that – as discussed in the previous section – the general equilibrium of the model implies that any exogenous shock that raises $w$ will increase $N, L$ and $X$ and can also lead to a capital inflow, if the right hand side of (24) rises sufficiently. It follows that the positive relationship between $w$ and $b$ in equation (16a) implies that a higher unemployment benefit rate will result in a higher level of economic activity. This is because in the presence of vertical linkages between sectors, the policy shock – by stimulating aggregate demand – triggers a virtuous circle of higher demand for intermediates and higher aggregate efficiency. Effectively, therefore, income redistribution from employed to unemployed factors contributes to alleviating the market inefficiencies that result from the positive externality of the mass of firms in the industry, thus leading to an increase in income. Moreover, if the potential aggregate scale economies are sufficiently strong (i.e. $\sigma$ is small), the offsetting of the policy shock on the government budget constraint will entail a reduction rather than an increase in the endogenous tax rate.

When the government offsets the policy shock by increasing the tax rate on the mobile factor, a capital outflow may occur. Figure 1 illustrates the main effects of the $\rho$-financed policy graphically.

Although the first part of the adjustment process is unaltered, the immediate increase in $\rho$ required to finance the policy may lead to a capital outflow that may more than compensate the initial inflow. This is more likely for small values of $\sigma$, when a high monopoly power of firms’ implies that, in response to the increase in the demand for $X$, entry will be limited and a small increase in $N$ will lead to a relatively large increase in the productivity of the intermediates (i.e. to large reduction in $P_x$). As a result, relative to a situation characterised by a larger value of $\sigma$, the extent of factor substitution towards the intermediate input will be higher, thus limiting the increase in the demand for and the resulting inflow of capital. However, we find that for most parameter values the increase in $b$ – by triggering the virtuous circle of higher wages, entry into the intermediate industry and higher income – will lead to a reduction in the tax rate on capital and to a capital inflow, that will therefore reinforce the positive process of cumulative causation.
In summary, our results show that regardless of the tax instrument used, a more generous welfare state (note that \(WB\) will typically increase despite the fall in unemployment) can lead to a higher welfare and to a capital inflow, even in those circumstances in which capital flows out of the country as a result of an increase in its tax rate.

4.2. Unions internalise the government budget constraint

When unions are large enough to internalise the link between taxation and unemployment benefits, the wage rate set by each union \(j\), given by equation (16b), will not be affected by the increase in \(b\). The policy multipliers are summarised in Table 2 below.

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<th>(\frac{\partial \ln \eta}{\partial b} )</th>
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(1) See Appendix A5 and A6 for derivations and analytical expressions. \(WB\) is the total unemployment benefit bill and \(Y_1^e\) is the excess supply of good 1.

(2) The sufficient condition for this multiplier to be positive is \(\sigma>2\). See Appendix A6.1.3 for details.

(3) A positive sign is obtained if \(\sigma\) is sufficiently large. See Appendix A6.1.3 for details.

(4) These signs are obtained by numerical simulations with plausible calibrations (see Appendix A6 for initial values and plots of the multipliers).

When the government offsets the policy shock by changing the tax rate on any of the immobile factors, there will be no effect on the economy. Using \(\rho\), however, will directly impinge on the availability of capital in the country and, except for the wage rate that does not change, the typical effects of the policy on the depth of the division of labour and on aggregate income are qualitatively the same as in the non-internalisation case. As \(\rho\) increases to finance the policy, a net capital outflow will occur (see Figure 2). The fall in \(K\) will increase the domestic interest rate and foster factor substitution towards \(X\) and \(Z\). The higher demand for \(X\) will trigger the virtuous circle of entry and higher aggregate efficiency discussed above, which will lead to an increase in higher income and to a larger high-tech sector. This will in turn raise the demand for \(K\) (more so if the high-tech sector is also relatively intensive in \(K\)) thus putting an upward pressure on the interest rate. For most parameter values, however, the increase in interest rate is

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not sufficient to offset the effects on $K$ of a higher $\rho$.

Hence, when unions internalise the budgetary effects of welfare state policies, whilst taxation of immobile factors does not affect economic performance, capital taxation can have a positive impact on the country’s income and on the extent of specialisation in the high-tech sector, despite the fact that a net capital outflow is likely to occur. It may be noted that in this latter case, the forces at work are qualitatively identical to that discussed in the previous subsection, but the extent of the final adjustments is weakened by the fact that the wage rate is unresponsive to changes in the policy variables.

4.3. A summary of the results

Our results so far suggest the following. First, by increasing the upstream sector’s general equilibrium wage, a higher welfare protection will deepen the division of labour within the economy and raise both aggregate income and the extent to which the country specialises in the high-tech sector$^{13}$. When the government finances the policy by taxing immobile factors, this virtuous circle is unambiguously reinforced by a capital inflow that increases the demand for the intermediate good and it may strengthen the shift of resources towards the high-tech good – to the extent that the latter is more intensive in the use of capital. Hence, contrary to the commonly held view, capital mobility (and the ensuing shrinking of the tax base) does not necessarily hinder the use of redistribution policies. Second, the use of capital taxation to offset the policy shock does not substantially alter the above results. A higher unemployment benefit, by leading to an increase in aggregate efficiency will improve welfare even when the larger tax rate on the returns to capital leads to a capital outflow. A higher capital taxation, however, does not always cause a capital outflow: a net inflow may occur that will reinforce the virtuous circle discussed above. On the whole, our findings cast doubt on the conventional wisdom discussed in the introduction.

5. NO CAPITAL MOBILITY

The analysis suggests that capital mobility enhances the virtuous circle stemming from aggregate scale economies and thus increases the positive impact of welfare state policies on economic performance. In order to gain more insights into its role, we now assume that capital is internationally immobile. The capital market resource constraint in (11) will now be given by

$^{13}$ Clearly, the expansionary effects of the policy reduce as the economy’s labour resource constraint tightens.
The equilibrium properties of the model are discussed in Appendix A4. It is tedious but straightforward to show that in this case the existence of a positive relationship between unemployment benefit and the depth of the division of labour within the economy is not unambiguous. A sufficient condition for $dN/db > 0$ can be found that requires the elasticity of substitution between intermediate varieties not to be ‘too large’ (see Appendix A6.2). In other words, without capital mobility, regardless of the tax instrument used to finance the policy shock, an increase in unemployment benefits will be more likely to have positive welfare effects the stronger is the extent of aggregate scale economies, i.e. the smaller is $\sigma$.

These results confirm that factor mobility needs not pose a threat to the sustainability of the welfare state and it might actually strengthen governments’ ability to finance it. This is because, by relaxing the resource constraint in the economy, capital mobility reduces the critical level of aggregate increasing returns at which a positive link between welfare state policies and the depth of the division of labour within the economy occurs.

6. CONCLUDING REMARKS

This paper has examined the role of economy-wide increasing returns to scale in shaping the relationship between welfare state policies and economic performance in a world with free trade in final goods and international capital mobility. Contrary to the conventional wisdom, we find that a retrenchment of welfare programmes is not an inevitable consequence of economic integration. Instead, by improving the exploitation of aggregate scale economies, social insurance policies and international openness may complement each other in increasing welfare.

These findings – which are consistent with and help explaining the evidence that goods and capital markets integration has not led to significant reductions in welfare states and tax burdens in OECD countries – crucially rest on the imperfectly competitive nature of the labour market and of the intermediate sector of the economy. In the former, unionisation results in equilibrium wages being increasing in the unemployment benefit and income tax rates. In the latter, monopolistic competition leads to the emergence of increasing returns to the range of available varieties. As a result, the expansionary effects of unemployment benefits and higher wages trigger a virtuous circle of entry in the intermediate sector, higher aggregate productivity, and higher income. We have shown that this virtuous circle is typically reinforced by capital mobility.
Unionisation plays an important role in the transmission mechanism between government policies and economic performance. Our results are broadly consistent with the relationship between unions’ monopoly power and degree of distortion of taxation found in the literature. However, a fundamental difference is that a higher degree of distortion resulting in a higher wage does not in our model imply a worsening of economic performance. On the contrary, it is when unions are strong enough to raise wages significantly but are not large enough to internalise the government budget constraint that an expansion of the welfare state will have a larger positive effect on aggregate welfare. This is because unions’ rent-seeking activity contributes, by increasing income, to trigger a virtuous circle that reduces the market failure associated with the sub-optimal provision of intermediate varieties. This result deserves closer scrutiny because it appears to counter the common perception that larger welfare states are typical of corporatist countries where ‘encompassing’ unions – by exchanging wage moderation for social protection – limit the distortionary effects of taxation. Our theoretical results can be taken to suggest that production externalities affect the impact of different labour market institutions on economic performance. Since there exists evidence (e.g. Caballero and Lyons, 1990) that countries exhibit different degrees of external economies of scale, a fruitful line for future research will be to study empirically how external economies affect the relationship between openness and the size of the public sector.

It is important to stress, however, that unionisation is not necessary for these results to emerge. Any form of labour market imperfection (e.g. efficiency wages) giving rise to a positive link between wages and policy instruments will lead to similar conclusions.

It is of course also true that welfare state policies are not the only way by which governments may trigger the virtuous process of cumulative causation described above. One lesson of economic policy is that intervention should be applied as closely as possible to the desired target. So, given that in this case the market imperfection consists of a sub-optimal production of varieties, industrial policy may well be more effective in correcting the distortion. This consideration, however, does not diminish the relevance of our analysis. The welfare state has played a specific social and political role in advanced industrial economies and attempts to retrench it are being met by opposition and may therefore result in a backlash against trade and capital markets liberalisation. It is therefore relevant to assess the extent to which openness and this type of policies are incompatible. Our results suggest that this needs not be the case.

Finally, we would like to point out that these conclusions are not qualitatively dependent on the specific model set up of the paper. As we show in Molana and Montagna (2002), the
fundamental forces at work in shaping the relationship between welfare state policies and economic performance are not altered if we relax the assumptions of a small open economy, the non-tradability of intermediates and the absence of unionisation in final good sectors.
Appendix

A1. Derivation of the wage elasticity of labour demand, $\varepsilon$, in equation (16)

Union $j$'s labour demand is $L_j^d = \int_{i \in N_j} l_i \, di$. Unions take account of firms' mark-up rule $p_{wN_j} = w_j$ and exploit the zero profit condition which implies $l_i = x_i$ where $x_i = \left( \frac{X^d}{P_x} \right)^{-\sigma}$, $i \in N_j$, and

$$X^d = \lambda_i P_i Y_i^x + \lambda_2 P_2 Y_2^x$$

is taken as given by the unions. Thus,

$$\frac{dL_j^d}{dw_j} = \int_{i \in N_j} \left[ \frac{\partial x_i}{\partial p_{i}} \frac{dp_{i}}{dw_j} + \frac{\partial x_i}{\partial P_x} \frac{dp_x}{dp_{i}} \frac{dp_{i}}{dw_j} \right] \, di.$$  \hspace{1cm} (A1.1)

Noting that $\frac{\partial x_i}{\partial p_{i}} = -\sigma \left( \frac{x_i}{p_i} \right)$, $\frac{\partial x_i}{\partial P_x} = (\sigma - 1) \left( \frac{x_i}{P_x} \right)$, the mark-up rule $p_{i} = w_j$ and hence $\frac{dp_i}{dw_j} = 1$, we obtain from (A1.1)

$$\frac{dL_j^d}{dw_j} = \int_{i \in N_j} \left( \frac{l_i}{w_j} \right) \left[ -\sigma + (\sigma - 1) \left( \frac{p_i}{P_x} \right) \frac{dp_x}{dp_{i}} \right] \, di.$$  \hspace{1cm} (A1.2)

In the symmetric equilibrium $L_j^d = \left( \frac{N}{J} \right) l_i$. In both the non-internalisation ($NI$) and the internalisation ($I$) cases $\frac{dP_x}{dp_i}$ is evaluated by partitioning $P_x$ in equation (5) as

$$P_x = \left( \int_{i \in N_j} \left( \frac{p_i}{P_x} \right)^{1-\sigma} \, di + \int_{i \in N_j} \left( \frac{p_i}{P_x} \right)^{-\sigma} \, di \right)^{\frac{1}{1-\sigma}}$$

which implies that $\frac{dP_x}{dp_i} = \int_{i \in N_j} \left( \frac{p_i}{P_x} \right)^{-\sigma} \, di$. In the symmetric equilibrium, the latter can be written as $\frac{dP_x}{dp_i} = \frac{N}{J} \left( \frac{p_i}{P_x} \right)^{-\sigma}$ and the price index will be given by $P_x = \left( \frac{1}{N^{1-\sigma}} \right) p_i$. These are used in (A1.2) to obtain

$$\frac{dL_j^d}{dw_j} = -\left( \frac{L_j^d}{w_j} \right) \left[ \sigma - (\sigma - 1) \frac{1}{J} \right]$$

and hence $\varepsilon = \sigma - (\sigma - 1) / J$ for both case ($NI$) and case ($I$). The only difference is therefore due to the number of unions which is much smaller in case ($I$).
A2. Derivation of the overall resource constraint, equation (24)

Using (8), (10), (11) and noting that \( X^s = X_1^d + X_2^d \), we obtain the following three resource constraints:

\[
\begin{align*}
X^s &= X_1^d + X_2^d = \lambda_1 \frac{P_i}{P_x} Y_1^s + \lambda_2 \frac{P_i}{P_x} Y_2^s, \\
Z &= Z_1^d + Z_2^d = \beta_1 \frac{P_i}{q} Y_1^s + \beta_2 \frac{P_i}{q} Y_2^s, \\
K^* + \bar{K} &= K_1^d + K_2^d = \alpha_1 \frac{P_i}{r} Y_1^s + \alpha_2 \frac{P_i}{r} Y_2^s.
\end{align*}
\]

(A2.1)

Solving the first two equations for \( Y_1^s \) and \( Y_2^s \) yields,

\[
P_i Y_1^s = \frac{\beta_2 P_i X - \lambda_2 q \bar{Z}}{\beta_2 \lambda_1 - \beta_1 \lambda_2},
\]

(A2.2)

and

\[
P_i Y_2^s = \frac{\lambda_2 q \bar{Z} - \beta_1 P_i X}{\beta_2 \lambda_1 - \beta_1 \lambda_2}.
\]

(A2.3)

Substituting these into the third equation yields the ‘overall’ resource constraint,

\[
r(\bar{K} + K^*) = \left( \frac{\alpha_2 \lambda_1 - \alpha_1 \lambda_2}{\beta_2 \lambda_1 - \beta_1 \lambda_2} \right) q \bar{Z} - \left( \alpha_2 \beta_1 - \alpha_1 \beta_2 \right) P_i X,
\]

(A2.4)

which is equation (24) in the text where we have defined parameters \( \theta_\lambda = (\alpha_2 \beta_1 - \alpha_1 \beta_2) / \Delta < 0 \), \( \theta_\mu = (\alpha_2 \lambda_1 - \alpha_1 \lambda_2) / \Delta > 0 \), and \( \Delta = \beta_2 \lambda_1 - \beta_1 \lambda_2 > 0 \) (the signs hold when \( \frac{\beta_1}{\beta_2} < 1 \leq \frac{\lambda_1}{\lambda_2} < \frac{\lambda_1}{\lambda_2} \)). This equation imposes a restriction on the values of the three factors and, regardless of the structure of factor intensities, should hold for all \( Y_1^s \) and \( Y_2^s \).

A3. Calibrating the model at the initial equilibrium.

The model consists of equations (15), (16) and (19)-(25) and the definition of total income and the degree of specialisation in trade measured by the excess supply of the high-tech good. Total income is given by equation (14) which, upon substitution from the government budget constraint in (13) yields

\[
M = wL + r \bar{K} + q \bar{Z} + \rho r K^*,
\]

(A3.1)

The excess supply of the high-tech good is \( P_i Y_1^c = P_i Y_1^s - P_i Y_1^d \) which, upon substitution from (A2.2) and equation (2) yields

\[
P_i Y_1^c = (\beta_2 P_i X - \lambda_2 q \bar{Z}) / \Delta - \mu M.
\]

(A3.2)
For ease of comparison, we calibrate the model at an initial equilibrium corresponding to the autarkic economy where \( K^* = 0, \ Y_1^* = (Y_1^* - Y_1^d) = 0, \) and \( Y_2^* = (Y_2^* - Y_2^d) = 0. \) We also assume that the tax rates are initially equal, \( \tau = \rho = \phi. \) It can be shown that replacing \( K^* = 0 \) and \( Y_2^* = (Y_2^* - Y_2^d) = 0 \) with \( K^* = cK \) and \( Y_2^* = (Y_2^* - Y_2^d) = \rho rcK \) will not change the results (\( c \) is some constant and the latter equation is obtained from the balance of payment equation in which \( Y_1^* = (Y_1^* - Y_1^d) = 0 \) and \( P_2 = 1 \) since good 2 is the numeraire). Using these initial values, we can solve equations (23)-(25) and (A3.1) and (A3.2) to obtain

\[
\begin{align*}
&P_X = wL \\
&qL = \psi wL \\
&rK = \xi wL \\
&b(L - L) = \tau(1 + \xi + \psi)wL \\
&M = (1 + \xi + \psi)wL
\end{align*}
\]

where \( \psi = \frac{\mu \beta_1 + (1 - \mu) \beta_2}{\mu \lambda_1 + (1 - \mu) \lambda_2} > 0 \) and \( \xi = \frac{\mu \alpha_1 + (1 - \mu) \alpha_2}{\mu \sigma_1 + (1 - \mu) \sigma_2} > 0. \)

Finally, we let the benefit rate to be set at the reservation wage, i.e. \( b = P\tilde{V} \), which yields:

\[
w^{vl} = \frac{2b}{(1 - \tau)(1 - \frac{1}{\epsilon})}, \quad \text{or} \quad w' = \frac{b}{(1 - \frac{1}{\epsilon})}. \tag{A3.4}
\]

### A4. Equilibrium properties of the model with no capital mobility.

In this case the model is equivalent to equations (16) and (19)-(25) where \( K^* = 0 \) is imposed. The relationship between \( w \) and \( N \) in this case can be obtained as follows. We solve equations (19)-(21) and set \( P_2 = 1 \) (treating good 2 as the numeraire) to obtain an equation for \( q \) in terms of \( N \) and \( w, \)

\[
q = P_1 \left[ \frac{A_1}{A_2} \right] [wN^{1-\sigma}]^{-\frac{\theta}{\sigma}}, \tag{A4.1}
\]

We also solve equations (23)-(25) to obtain an equation for \( q \) in terms of \( wL, \)

\[
q = \frac{b(L - L) - (\tau - \rho \theta_3)wL}{(\phi + \rho \theta_3)\tilde{Z}}. \tag{A4.2}
\]

Equating the right-hand-sides of (A4.1) and (A4.2) yields

\[
c \left[ wN^{1-\sigma} \right] = b(L - L) - (\tau - \rho \theta_3)wL \tag{A4.3}
\]
where \( c = (\phi + \rho \theta_i) P \left( \frac{\theta_i \theta_j}{\theta_i + \theta_j} \right) > 0 \) and \( \theta = \frac{\theta_i}{\theta_j} > 0 \). Noting that from (22) \( L = \gamma \sigma N \), (A4.3) gives a relationship between \( w \) and \( N \) which should hold in equilibrium. Differentiating (A4.3) totally, taking account of (22) and letting \( \rho = \tau \) yields

\[
\frac{dN}{dw} = \left( \frac{N}{w} \right) \left[ \frac{\theta c}{\sigma - 1} \left( w^{\frac{1}{\sigma}} \right)^\theta - (bL + (1 - \theta_x) \omega L) \right] = \left[ (1 + \psi + \xi) - (1 - \theta_x) \right] - \frac{b}{\omega} - (1 - \theta_x) > 0
\]

The denominator of the above is always positive. Using (A4.3) and taking account of the calibration in (A3.3), the condition for the numerator to be positive is

\[
\left( \frac{\theta}{\sigma - 1} \right) \left[ (1 + \psi + \xi) - (1 - \theta_x) \right] = \frac{b}{\omega} - (1 - \theta_x) > 0
\]

Finally, we can use (A3.4) to replace \( b/w \) in the above expression, for each case \((NI)\) and \((I)\), and obtain the condition that the parameters \((\theta, \theta_x, \xi, \psi, \sigma)\), the union size, and the initial tax rate \( \tau \) should satisfy when signing the multipliers derived below.

A5. **Linearising the equations of the model.**

Starting from the initial equilibrium described above, we study the qualitative effects of a change in \( b \) financed by one of the tax rates. For simplicity, whenever possible, we linearise the model in terms of proportional changes in variables. In the following, a change and a proportional change are denoted by a dot and a hat \((^\hat{\cdot})\) over a variable respectively. Totally differentiating equations (15), (16), (19) to (25), (A3.1) and (A3.2) we obtain

\[
\hat{r} - \left( \frac{\rho}{1 - \rho} \right) \hat{\rho} = \hat{r}^\ast \quad (A5.1)
\]

\[
\eta b \hat{b} - \left[ w(1 - \eta \tau) \left( 1 - \frac{1}{\epsilon} \right) \right] \hat{w} - \left( \frac{\eta \tau}{1 - \eta \tau} \right) \hat{\tau} = 0 \quad (A5.2)
\]

\[
\hat{P}_\xi = \theta_x \hat{r} \quad (A5.3)
\]

\[
\hat{q} = -\theta_x \hat{r} \quad (A5.4)
\]

\[
\hat{P}_\pi = \frac{1}{1 - \sigma} \hat{N} + \hat{w} \quad (A5.5)
\]

\[
\hat{L} = \hat{N} \quad (A5.6)
\]

\[
\hat{P}_\xi + \hat{X} = \hat{w} + \hat{L} \quad (A5.7)
\]
\[ r\dot{K}^* + r(\dot{K} + K^*) = \theta_x(q\dot{Z})q - \theta_x(P_xX)(\dot{P}_x + \dot{X}) \]  
(A5.8)

\[ \tau wL(\dot{\tau} + \dot{\hat{w}} + \dot{\hat{L}}) + \phi q\dot{Z}(\dot{\phi} + \dot{\hat{q}}) + \rho r(\dot{K} + K^*)(\dot{\rho} + \dot{\hat{r}}) + \rho rK^* + bL\dot{\hat{L}} - b(\dot{\tau} - L)\dot{\hat{b}} = 0 \]  
(A5.9)

\[ \dot{M} = wL(\dot{\hat{L}} + \dot{\hat{w}}) + r\dot{\hat{K}} + q\dot{\hat{Z}}q + \rho r\dot{K^*} \]  
(A5.10)

\[ \ddot{Y}^*_1 = \beta_2 P_xX(\dot{P}_x + \dot{X}) - \lambda_2 q\dot{Z}\dot{q} \]  
\[ \frac{\mu M}{P_1} \]  
(A5.11)

These equations are solved to derive the policy multipliers.

**A6. The policy multipliers due to a change in \( b \).**

For ease of exposition, where possible we present the proportional change rather than the change. Recall that a change and a proportional change are denoted by a ‘dot’ and a ‘hat’ (^) over a variable, respectively. \( WB \) denotes the welfare bill, \( WB = b(L - L) \) and in all the expressions \( \tau \) is the initial tax rate, given that initially all tax rates are set equal, i.e. \( \rho = \phi = \tau \).

Finally, due to the complexity of signing the multipliers in some cases we have resorted to numerical simulations. In these cases, we have given the three dimensional graphs which plot the multipliers against the crucial parameters \( \sigma \) and \( \tau \), using the following calibration:

\[ \alpha_1 = 0.3; \quad \alpha_2 = 0.3; \quad \beta_1 = 0.2; \quad \beta_2 = 0.5; \quad \lambda_1 = 0.5; \quad \lambda_2 = 0.2; \quad \mu = 0.25; \quad r^* = 0.05; \quad b = \bar{b} = 1; \quad \bar{L} = 1; \quad J^{NL} = 100; \quad J^I = 10; \]

**A6.1. Model with capital mobility**

**A6.1.1. Policy financed using the tax rate \( \tau \)**

**Case (NI): Non-internalisation**

\[ \frac{\dot{P}_x}{\dot{b}} = 0; \quad \frac{\dot{q}}{\dot{b}} = 0; \quad \frac{\dot{r}}{\dot{b}} = 0; \quad \frac{\dot{\hat{N}}}{\dot{b}} = \frac{e(\sigma - 1)[1 + \tau(1 + 2(\xi + \psi))]}{(1 - \tau)(\epsilon + 1) + \sigma(\epsilon - 1) + \tau(1 + \epsilon(1 - 2\theta_\chi))} > 0; \]

\[ \frac{\dot{\hat{w}}}{\dot{b}} = \left( \frac{1}{\sigma - 1} \right) \frac{\dot{\hat{N}}}{\dot{b}} > 0; \quad \frac{\dot{\hat{L}}}{\dot{b}} = \frac{\dot{\hat{N}}}{\dot{b}} > 0; \quad \frac{\dot{\hat{X}}}{\dot{b}} = \left( \frac{\sigma}{\sigma - 1} \right) \frac{\dot{\hat{N}}}{\dot{b}} > 0; \]

\[ \frac{\dot{\hat{M}}}{\dot{b}} = \left( \frac{2e\sigma\bar{L}(1 - \tau\theta_\chi)}{(\sigma - 1)[(\epsilon - 1) + \tau(1 + \epsilon(1 + 2(\xi + \psi)))]} \right) \frac{\dot{\hat{N}}}{\dot{b}} > 0; \]

\[ \frac{\dot{\hat{K}}^*}{\dot{b}} = \left( \frac{-2e\sigma\bar{L}(1 - \tau\theta_\chi)}{(\sigma - 1)[(\epsilon - 1) + \tau(1 + \epsilon(1 + 2(\xi + \psi)))]} \right) \frac{\dot{\hat{N}}}{\dot{b}} > 0. \]

The necessary condition for \( \frac{\dot{\hat{c}}}{\dot{b}} = \left( \frac{2e\sigma\bar{L}[b_2 - (1 - \tau\theta_\chi)\mu]}{(\sigma - 1)[(\epsilon - 1) + \tau(1 + \epsilon(1 + 2(\xi + \psi)))]} \right) \frac{\dot{\hat{N}}}{\dot{b}} > 0 \) is
\[ 0 < \mu < \frac{\beta_2}{(1 - \tau \theta_1)\Delta} \quad \text{and the sufficient condition for} \quad \frac{\dot{\tau}}{b} = \frac{(e - 1)(\sigma - 1)(1 - \tau) - 2e[2(1 + \zeta + \psi) - \sigma(1 - \tau \theta_1)]}{2(\sigma - 1)\epsilon[1 + \tau(1 + 2(\zeta + \psi))]} \quad \frac{\dot{N}}{b} > 0 \quad \text{is} \quad 2(1 + \zeta + \psi) - \sigma(1 - \tau \theta_1) < 0. \]

**Case (I): Internalisation**

All multipliers are zero in this case and \( \frac{\dot{\tau}}{b} = 1 + \zeta + \psi > 0. \)

**A6.1.2. Policy financed using the tax rate \( \phi. \)**

**Case (NI): Non-internalisation**

\[
\begin{align*}
\frac{\dot{P}}{b} &= 0; \quad \frac{\dot{q}}{b} = 0; \quad \frac{\dot{r}}{b} = 0; \quad \frac{\dot{N}}{b} = \frac{\sigma - 1}{2} > 0; \quad \frac{\dot{\psi}}{b} = \frac{1}{2}; \quad \frac{\dot{L}}{b} = \frac{\sigma - 1}{2} > 0; \quad \frac{\dot{X}}{b} = \frac{\sigma}{2} > 0;
\end{align*}
\]

\[
\frac{K^*}{b} = \frac{-\theta_1(1 - \tau)\sigma \bar{L}}{[(e - 1) + \tau(1 + 2(\zeta + \psi))]^{\frac{1}{2}}} > 0; \quad \frac{M}{b} = \frac{(1 - \tau \theta_1)\sigma \epsilon \bar{L}}{(e - 1) + \tau(1 + 2(\zeta + \psi))^{\frac{1}{2}}} > 0.
\]

The necessary condition for \( \frac{\dot{Y}_1^e}{b} = \frac{(\beta_2 - (1 - \tau \theta_1)\mu)\sigma e}{[(e - 1) + \tau(1 + 2(\zeta + \psi))]^{\frac{1}{2}}} > 0 \) is \( 0 < \mu < \frac{\beta_2}{(1 - \tau \theta_1)\Delta}, \)

and the sufficient condition for

\[
\frac{\dot{\phi}}{b} = \frac{1 + 2(\zeta + \psi) + \sigma \theta_1}{2\psi} - \frac{(\sigma - 1)(2e - (e + 1)(1 - \tau))}{4\psi \theta_1} < 0 \quad \text{is} \quad 1 + 2(\zeta + \psi) + \sigma \theta_1 < 0.
\]

**Case (I): Internalisation**

All multipliers are zero in this case and \( \frac{\dot{\phi}}{b} = \frac{1 + \zeta + \psi}{\psi} > 0. \)

**A6.1.3. Policy financed using the tax rate \( \rho. \)**

**Case (NI): Non-internalisation**

\[
\begin{align*}
\frac{\dot{w}}{b} &= \frac{1}{2}; \quad \frac{\dot{L}}{b} = \frac{\dot{N}}{b}; \quad \frac{\dot{X}}{b} = \left(\frac{\sigma}{\sigma - 1}\right) \frac{\dot{N}}{b} \quad \text{and the sufficient conditions} \quad (i) \quad 1 - \tau(2 + \theta_1) > 0 \quad \text{and} \quad (ii) \quad 1 + \tau[2(\psi + \xi) + \theta_1] > 0 \quad \text{are required for}
\end{align*}
\]

\[
\frac{\dot{N}}{b} = \frac{-\sigma - 1\epsilon[\psi \theta_1(1 - \tau(2 + \theta_1)) - \theta_1(1 + (2(\psi + \xi) + \theta_1)\tau)]}{\theta_1[(\sigma - 1)(e - 1 + \tau(1 + e(1 - 2\theta_1)))] + 2e(1 - \tau) - 2e\psi \theta_1(1 - \tau(2 + \theta_1))} > 0.
\]
Also, $$\frac{\hat{P}}{b} = A_p \frac{\hat{N}}{b}, \quad \frac{\hat{r}}{b} = \left(\frac{A_p}{\theta_x}\right) \frac{\hat{N}}{b}, \quad \frac{\hat{q}}{b} = \left(-\frac{\theta_x A_p}{\theta_x}\right) \frac{\hat{N}}{b}, \quad \frac{\hat{\rho}}{b} = \frac{(1-\tau)A_p}{\tau \theta_x} \frac{\hat{N}}{b},$$ where the necessary conditions for

$$A_p = \frac{-\theta_x \left(\varepsilon - 1 + \tau \left(1 + \varepsilon (3 + 3(\psi + \xi))\right) - \delta \left(\varepsilon - 1 + \tau \left(1 + \varepsilon (1 - 2\theta_z)\right)\right)\right)}{\theta_x \left[1 + \tau (2(\psi + \xi) + \theta_x)\right] - \psi \theta_x \left(1 - \tau (2 + \theta_z)\right)} > 0$$

are

$$\sigma < \frac{\varepsilon - 1 + \tau (1 + \varepsilon (3 + 3(\psi + \xi)))}{\varepsilon - 1 + \tau (1 + \varepsilon (1 - 2\theta_z))}$$

and conditions (i) and (ii) above, in which case, $$\frac{\hat{P}}{b} < 0,$$

$$\frac{\hat{r}}{b} > 0, \quad \frac{\hat{q}}{b} < 0, \quad \text{and} \quad \frac{\hat{\rho}}{b} > 0.$$ Given the complexity of signing the multipliers in this case, the following graphs provide further indication.

**Figure 1 – Effects of a ρ-financed increase in b: case (NI), ‘non-internalisation’**
Case (I): Internalisation

\[
\frac{\dot{w}}{b} = 0, \text{ and } \sigma > 2 \text{ is the sufficient condition for } \frac{\dot{L}}{b} = \frac{\dot{N}}{b} > 0; \quad \frac{\dot{X}}{b} = \left(\frac{\sigma}{\sigma - 1}\right) \frac{\dot{N}}{b} > 0;
\]

\[
\frac{\dot{Q}}{b} = \left(\frac{-1}{\sigma - 1}\right) \frac{\dot{N}}{b} < 0; \quad \frac{\dot{R}}{b} = \left(\frac{\theta_x}{\theta_x (\sigma - 1)}\right) \frac{\dot{N}}{b} < 0; \quad \frac{\dot{r}}{b} = \left(\frac{-1}{\theta_x (\sigma - 1)}\right) \frac{\dot{N}}{b} > 0; \quad \text{and}
\]

\[
\frac{\dot{R}^*}{b} = \frac{\theta_x x_0 (\sigma - 1)(1 + \xi + \psi)}{\theta_x (\sigma - 1) r (e - 1 + \tau(1 - \theta_x))} \frac{\dot{N}}{b} < 0. \quad \text{The sufficient condition}
\]

for \[
\frac{\dot{K}}{b} = \left(\frac{(1 - \tau) e_L (\psi \theta_x (1 + \theta_x) - \theta_x (1 + \theta_x (\sigma - 1)))}{\theta_x (\sigma - 1) r (e - 1 + \tau(1 + \xi + \psi))}\right) \frac{\dot{N}}{b} < 0 \quad \text{is} \quad 2 < \sigma < 1 - \frac{1}{\theta_x}.
\]

are \[
\frac{\dot{M}}{b} = \left(\frac{-e_L (-\sigma \theta_x - \tau(\psi \theta_x (1 + \theta_x) - \theta_x (1 + \theta_x (\sigma - 1))))}{\theta_x (\sigma - 1) (e - 1 + \tau(1 + \xi + \psi))}\right) \frac{\dot{N}}{b};
\]

\[
\frac{\dot{Y}}{b} = C (-(\sigma \theta_x - \tau(\psi \theta_x (1 + \theta_x) - \theta_x (1 + \theta_x (\sigma - 1)))) \Delta \mu - (\psi \theta_x \lambda \xi - (\sigma - 1) \theta_x \beta) \dot{N} \frac{b}{b},
\]

where \[
C = \frac{-e_L}{\theta_x (\sigma - 1) \Delta (e - 1 + \tau(1 + \xi + \psi))} > 0; \quad \text{and } \frac{\dot{\mu}}{b} = \left(\frac{-1 - \tau}{\theta_x (\sigma - 1) \tau}\right) \frac{\dot{N}}{b} > 0.
\]

Given the complexity of the signing of the multipliers in this case, following graphs provide further indication.
Figure 2 – Effects of a $\rho$-financed increase in $b$: case (I), ‘internalisation’

A6.2. Model without capital mobility

A6.2.1. Policy financed using the tax rate $\tau$.

Case (NI): Non-internalisation

The necessary condition for

$$\frac{\dot{N}}{b} = \frac{\varepsilon(\sigma-1)[1+\tau(1+2(\xi+\psi))][\psi\theta_x(1+\theta_x)-\theta_x(1-\theta_x)]}{2\varepsilon[\psi\theta_x(1+\theta_x)+\theta_x(1-\tau\theta_x)]-(1-\tau)[(\varepsilon+1)(\sigma-1)[\psi\theta_x(1+\theta_x)-\theta_x(1-\theta_x)]]} > 0$$

is

$$\frac{\sigma}{\sigma-1} > \frac{(1-\tau)(\varepsilon+1)[\psi\theta_x(1+\theta_x)-\theta_x(1-\theta_x)]}{2\varepsilon[\psi\theta_x(1+\theta_x)+\theta_x(1-\tau\theta_x)]}, \text{ i.e. } \sigma \text{ sufficiently small.}$$

If $\frac{\dot{N}}{b} > 0$, then

$$\frac{\dot{L}}{b} = \frac{\dot{N}}{b} > 0; \quad \frac{\dot{X}}{b} = \left(\frac{\sigma}{\sigma-1}\right)\frac{\dot{N}}{b} > 0; \quad \frac{\dot{M}}{b} = \left(\frac{2\varepsilon\sigma L}{(\sigma-1)(\varepsilon-1)(1+\tau(1+e(1+2(\xi+\psi)))]}\right)\frac{\dot{N}}{b} > 0;$$
\[
\frac{\dot{P}}{b} = \left( \frac{-\sigma \theta^2}{(\sigma - 1)[\psi \theta_x (1 + \theta_x) - \theta_x (1 - \theta_x)]} \right) \frac{\hat{N}}{b} < 0; \quad \frac{\dot{q}}{b} = \left( \frac{\sigma \theta_x \theta_z}{(\sigma - 1)[\psi \theta_x (1 + \theta_x) - \theta_x (1 - \theta_x)]} \right) \frac{\hat{N}}{b} < 0;
\]

\[
\dot{\tau} = \left( \frac{-\sigma \theta}{(\sigma - 1)[\psi \theta_x (1 + \theta_x) - \theta_x (1 - \theta_x)]} \right) \frac{\hat{N}}{b} > 0; \quad \text{and the sufficient condition for}
\]

\[
\dot{w} = \frac{(\psi \theta_x (1 + \theta_x) - \theta_x (1 + \theta_x (\sigma - 1)))}{(\sigma - 1)[\psi \theta_x (1 + \theta_x) - \theta_x (1 - \theta_x)]} \frac{\hat{N}}{b} > 0 \quad \text{is} \quad \sigma < 1 - \frac{1}{\theta_x}.
\]

Finally, \( \mu < \frac{(\psi \theta_x (1 + \theta_x) - \theta_x \beta_x \psi \lambda_z)}{\Delta[\psi \theta_x (1 + \theta_x) - \theta_x (1 - \theta_x)]} \) is the necessary condition for

\[
\frac{\dot{y}^c}{b} = C((\psi \theta_x (1 + \theta_x) - \theta_x \beta_x \psi \lambda_z) - \Delta[\psi \theta_x (1 + \theta_x) - \theta_x (1 - \theta_x)]\mu) \frac{\hat{N}}{b} > 0, \quad \text{where}
\]

\[
C = \frac{2\epsilon \alpha \bar{\tau}}{\Delta(\sigma - 1)[(\epsilon - 1) + \tau(1 + \epsilon(1 + 2(\xi + \psi)))][\psi \theta_x (1 + \theta_x) - \theta_x (1 - \theta_x)]} > 0.
\]

Given the complexity of the signing of the multipliers in this case, following graphs provide further indication.

**Figure 3 – Effects of a \( \tau \)-financed increase in \( b \): case (NI), ‘non-internalisation’**
Case (I): Internalisation

All multipliers are zero in this case and \( \dot{\hat{\tau}} / \hat{b} = 1 + \hat{\xi} + \psi > 0 \).

A6.2.2. Policy financed using the tax rate \( \phi \).

Case (NI): Non-internalisation

\[ \frac{\hat{\dot{w}}}{b} = \frac{1}{2}, \] and the sufficient condition for \( \frac{\hat{N}}{b} = \frac{(\sigma - 1)[\psi \theta_x (1 + \theta_z) - \theta_x (1 - \theta_x)]}{2[\psi \theta_z (1 + \theta_z) - \theta_x (1 + (\sigma - 1) \theta_x)]} > 0 \) is \( \sigma < 1 - \frac{1}{\theta_x} \).

Provided that this condition holds, then \( \frac{\hat{\dot{L}}}{b} = \frac{\hat{N}}{b} > 0; \quad \frac{\hat{\dot{X}}}{b} = \left( \frac{\sigma}{\sigma - 1} \right) \frac{\hat{N}}{b} > 0; \)

\[ \frac{\hat{\dot{M}}}{b} = \left( \frac{2\epsilon \sigma \xi}{(\sigma - 1)[(\epsilon - 1) + \tau (1 + \epsilon (1 + 2 (\xi + \psi)))]} \right) \frac{\hat{N}}{b} > 0; \]

\[ \frac{\hat{\dot{P}}_x}{b} = \left( \frac{-\sigma \theta_x^2}{(\sigma - 1)[\psi \theta_x (1 + \theta_z) - \theta_x (1 - \theta_x)]} \right) \frac{\hat{N}}{b} < 0; \quad \frac{\hat{q}}{b} = \left( \frac{\sigma \theta_x \theta_z}{(\sigma - 1)[\psi \theta_x (1 + \theta_z) - \theta_x (1 - \theta_x)]} \right) \frac{\hat{N}}{b} > 0; \]

and \( \frac{\hat{\dot{r}}}{b} = \left( \frac{-\sigma \theta_x}{(\sigma - 1)[\psi \theta_x (1 + \theta_z) - \theta_x (1 - \theta_x)]} \right) \frac{\hat{N}}{b} > 0. \)
Finally, \( \mu < \left( \psi \theta_x (1 + \theta_x) - \theta_x \right) \beta_z - \theta_x \psi \lambda_x \) is the necessary condition for

\[
\frac{\ddot{y}_1}{b} = C((\psi \theta_x (1 + \theta_x) - \theta_x) \beta_z - \theta_x \psi \lambda_x \psi \lambda - \Delta [\psi \theta_x (1 + \theta_x) - \theta_x (1 - \theta_x)] \mu) \frac{\hat{N}}{b} > 0,
\]

where

\[
C = \frac{2\epsilon \sigma \vec{L}}{\Delta (\sigma - 1) (1 + \epsilon (1 + 2(\xi + \psi))) [\psi \theta_x (1 + \theta_x) - \theta_x (1 - \theta_x)]} > 0.
\]

Given the complexity of the signing of the other multipliers in this case, following graphs provide some indication.

**Figure 4 – Effects of a \( \phi \)-financed increase in \( b \): case (NI), ‘non-internalisation’**

![Graphs](image)

**Case (I): Internalisation**

All multipliers are zero in this case and \( \frac{\hat{\phi}}{b} = \frac{1 + \xi + \psi}{\psi} > 0. \)

**A6.2.3. Policy financed using the tax rate \( \rho \).**

**Case (NI): Non-internalisation**

\( \frac{\hat{\lambda}}{b} = \frac{1}{2} \), and the sufficient condition for

\[
\frac{\hat{N}}{b} = \frac{(\sigma - 1)[\psi \theta_x (1 + \theta_x) - \theta_x (1 - \theta_x)]}{2[\psi \theta_x (1 + \theta_x) - \theta_x (1 + (\sigma - 1) \theta_x)]} > 0 \quad \text{is} \quad \sigma < 1 - \frac{1}{\theta_x}.
\]

Provided that this condition holds, then

\[
\frac{\hat{L}}{b} = \frac{\hat{N}}{b} > 0; \quad \frac{\hat{X}}{b} = \left( \frac{\sigma}{\sigma - 1} \right) \frac{\hat{N}}{b} > 0;
\]

\[
\frac{\hat{M}}{b} = \left( \frac{2\epsilon \sigma \vec{L}}{(\sigma - 1)(1 + \epsilon (1 + 2(\xi + \psi)))} \right) \frac{\hat{N}}{b} > 0; \quad \frac{\hat{P_z}}{b} = \left( \frac{-\sigma \theta_z^2}{(\sigma - 1)[\psi \theta_x (1 + \theta_x) - \theta_x (1 - \theta_x)]} \right) \frac{\hat{N}}{b} < 0;
\]
\[
\frac{\dot{q}}{b} = \left(\frac{\sigma\theta_2}{(\sigma - 1)\psi_0(1 + \theta_2) - \theta_x(1 - \theta_1)}\right) \frac{\hat{N}}{b} < 0; \text{ and } \frac{\dot{r}}{b} = \left(\frac{-\sigma\theta_1}{(\sigma - 1)\psi_0(1 + \theta_1) - \theta_x(1 - \theta_2)}\right) \frac{\hat{N}}{b} > 0.
\]

Finally, \( \mu < \frac{(\psi_0(1 + \theta_2) - \theta_x\theta_2\sigma\lambda)}{\Delta[\psi_0(1 + \theta_2) - \theta_x(1 - \theta_1)]} \) is the necessary condition for

\[
\frac{\dot{Y}_1^c}{b} = C(\psi_0(1 + \theta_2) - \theta_x\beta_2 - \theta_x\psi_0\lambda_2 - \Delta[\psi_0(1 + \theta_2) - \theta_x(1 - \theta_1)]\mu) \frac{\hat{N}}{b} > 0, \text{ where}
\]

\[
C = \frac{2\epsilon\sigma\xi}{\Delta(\sigma - 1)(\epsilon - 1) + \tau(1 + \epsilon(1 + 2(\xi + \psi)))}\psi_0(1 + \theta_2) - \theta_x(1 - \theta_1) > 0.
\]

Given the complexity of the signing of the other multipliers in this case, following graphs provide some indication.

**Figure 4 – Effects of a \( \rho \)-financed increase in \( b \): case (NI), ‘non-internalisation’**

**Case (I): Internalisation**

All multipliers are zero in this case and

\[
\frac{\hat{\rho}}{b} = \frac{1 + \xi + \psi}{\psi_0 - \theta_i} > 0.
\]
References


