Efficiency Wages, Unemployment and Macroeconomic Policy

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ABSTRACT:

We construct a stylised macro-model with goods and labour market imperfections to show that the economy can rationally operate at an inefficient, or ‘low-effort’, equilibrium in the neighbourhood of which the relationship between output and unemployment is, in contrast to Okun’s Law, positive. We use the Kalman-filter approach allowing for trends, cyclical changes and breaks to examine data from the G7 countries over period 1960-2001, and find that only German data strongly favour a persistent negative relationship between the level of output and rate of unemployment. Our results suggest that circumstances exist in which market imperfections pose serious obstacles to the smooth working of expansionary and/or stabilization policies.

KEYWORDS:

Efficiency wages; effort supply; monopolistic competition; multiple-equilibria; fiscal multiplier; Kalman filter;

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1. **Introduction**

In the last few decades, industrialised nations have been subjected to a variety of external and policy-induced demand shocks while simultaneously experiencing significant changes in their labour productivity and employment. Meanwhile, governments have been concerned to maintain a balance between implementing those policies which protect workers against job losses by reducing the hardship of unemployment and those which restrain the unemployment rate. However, as Lindbeck (1992) warns, unless we have a clear understanding of how such policies work, their implementation may produce unexpected consequences: *In the context of a nonmarket-clearing labour market, it is certainly reasonable to regard unemployment, in particular highly persistent unemployment, as a major macroeconomic distortion. There is therefore a potential case for policy actions, provided such actions do not create more problems than they solve. Experience in many countries suggests that the latter reservation is not trivial."

In this paper we focus on one such case by examining the relationship between the level of output and the rate of unemployment. The common belief regarding this relationship is dominated by Okun’s Law, which predicts a negative relationship between changes in these variables. Our aim is to explore whether it is possible for an economy to deviate from this in a systematic way. More specifically, we ask whether there are circumstances in which a rise in the rate of unemployment can lead to an increase the level of output. We develop a theoretical model that shows such a result can be obtained when labour and goods markets operate under certain (plausible) conditions. Clearly, such departures from ‘standard’ results are expected when models are modified to deviate from competitive markets by allowing for particular types of rigidities or distortions. Typical examples in the related literature are the introduction of ‘efficiency wages’, ‘unionisation’, ‘wage contracts’, and ‘unemployment insurance’. However, as far as we are aware, only studies on the latter topic have reported
results which show that changes in output and unemployment can be positively correlated. For instance, Acemoglu and Shimer (2000) focus on the effect of raising unemployment insurance within a search model and conclude that more generous welfare programmes can in fact raise output and welfare despite giving rise to a higher unemployment.

Given that we are interested in examining the effect of a typical macroeconomic policy – e.g. the rise in government expenditure financed by taxes – in the presence of relevant market imperfections, in this paper we construct a model in the tradition of Blanchard and Kiyotaki (1987). The model allows for a distortion in the labour market through incorporating a variant of the efficiency wage hypothesis whereby involuntary unemployment gives rise to externalities that could be exploited by economic agents; price-setting firms use high or rising unemployment as a device to deter shirking. The novelty of the variant used in this paper is that, unlike the existing models in which a worker’s effort level is discrete and can assume either a low or a high value, it allows a worker’s optimal effort supply to be a continuous function of its determinants. These determinants include the real wage, unemployment insurance and rate of unemployment in the economy. In such circumstances the supply side is shown to exhibit a non-linearity which is adequately captured by a humped-shape relationship between output and unemployment rate. It follows that the economy can, at any point in time, be in one of the three possible states with regard to the effort level. The standard case, in line with Okun’s Law in which output and unemployment rate are negatively related, occurs in the ‘high-effort’ state where the economy can be said to be operating ‘efficiently’. In this case, to raise the level of output in response to a rise in aggregate demand firms need to employ more workers. The opposite case occurs in the ‘low-effort’ state in which the economy may be said to be operating ‘inefficiently’. In this situation a higher level of output can be achieved at a lower level of employment since firms find it more profitable to meet the rise in demand by inducing the
workers to raise their (optimal) effort supply. These two states are separated by a third, the ‘threshold effort’ state, which corresponds to the peak of the humped-shaped relationship where the combination of employment and effort yields the maximum level of output. In this sense, therefore, in the ‘threshold effort’ state the economy may be said to be operating without any slack despite the existence of a positive level of ‘involuntary’ unemployment. Clearly, such an economy may experience multiple-equilibria. We show that when the economy is trapped in a ‘low-effort’ equilibrium, positive demand shocks can lead, perversely, to an increase in unemployment.

To explore the extent to which the non-linearity predicted by the model is supported by evidence, we examine the empirical relationship between unemployment rate and level of output for data from the G7 countries. Our empirical analysis is based on estimating a state space ‘local linear trend’ model using the Kalman-filter. This approach allows us to account both for secular and cyclical variations and for changes in productivity of other factors, which do not explicitly feature in the analysis. Our evidence suggests that whilst ‘low-effort’ periods have occurred significantly within the sample, periods corresponding to ‘threshold effort’ seem to dominate and only German data shows a strong support for more frequent occurrence of the ‘high-effort’ case.

The rest of the paper proceeds as follows. Section 2 outlines the model and shows how the non-linearity described above emerges and derives the typical fiscal policy multiplier to illustrate the perverse policy effect. Section 3 explains our econometric method and reports the evidence for each of the G7 countries and Section 4 concludes the paper. The

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1 Other recent studies which examine the link between unemployment and productivity include Malley and Moutos (2001), Leith and Li (2001), Daveri and Tabellini (2000), Blanchard (1998), Caballero and Hammour (1998a,b), Gordon (1997a) and Manning (1992). However, none of these studies explores the link between unemployment and output arising from both labour and product market imperfections.
Appendix outlines the derivation of certain results, which are not explained within the main body of the paper.

2. **The Relationship between Output and Unemployment: Theory**

In this section we construct and analyse a stylised theoretical macro-model which is in the tradition of Blanchard and Kiyotaki (1987) but allows the economy to sustain some level of unemployment in equilibrium. More specifically, we endow the model with market imperfections in both the labour and the goods market by assuming that monopolistically competitive firms reward workers’ effort by paying efficiency wages.

Following the work of Shapiro and Stiglitz (1984) and Yellen (1984), a number of models have employed some version of the efficiency wage hypothesis to study various aspects of macroeconomic activity. Examples can be found in: Agénor and Aizenman (1999) and Rebitzer and Taylor (1995) on fiscal and labour market policies; Andersen and Rasmussen (1999), Pisauro (1991) and Carter (1999) on the role of the tax system; Leamer (1999) on specialisation; Albrecht and Vroman (1996), Fehr (1991) and Smidt-Sørensen (1990) on properties of labour demand; and Smidt-Sørensen (1991) on working hours. In this paper we employ a standard version of the hypothesis which postulates that workers can adjust their effort supply in response to the wage and threat of losing their job. But rather than using a discrete choice between low and high effort levels we allow for the optimal effort supply to be a continuous function of its determinants.

The model is static and describes an economy with three types of agents: firms, households and a government. Firms are monopolistically competitive and each firm produces one variety of a horizontally differentiated product using labour as input with an increasing returns to scale technology. Households are endowed with a unit of labour, which
they supply inelastically. Unemployed households receive a benefit transfer from the government. The government revenue, raised by taxing the households, is used to subsidise the unemployed and to pay for government consumption. The final good in the economy is the Dixit-Stiglitz CES bundle of horizontally differentiated varieties.

The demand side of the model consists of the households’ and government’s consumption. The latter is given by

$$\int_{j=1}^{N} P_j g_j \, dj = PG, \quad (1)$$

where $j$ is the index denoting a variety of the differentiated good, $N$ is the mass of varieties on offer and $P_j$ and $g_j$ are the price and quantity of variety $j$. It is straightforward to show that

$$g_j = \left( \frac{P_j}{P} \right)^{-\frac{1}{s}} \frac{G}{N}, \quad (2)$$

where $G$ is the corresponding CES bundle

$$G = \left( N^{-\frac{1}{s}} \int_{j=1}^{N} g_j^{-\frac{1}{(1-s)}} \, dj \right)^{1/[1-(1/s)]}.$$ 

Note that the constant elasticity of substitution\(^2\) between any two varieties is given by $s>1$; $P$ is the price index dual to $G$; and (2) maximises $G$ above subject to (1).

The government expenditure comprises (1) and the unemployment benefit payments $B$ per unemployed worker/household and is financed by a lump sum tax\(^3\), $T$. Normalising the number of households to unity and denoting the proportion of unemployed households by $u$, the government budget constraint is written as

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\(^2\) See Molana and Zhang (2001) for a study of the role of a variable elasticity of substitution, in the context of fiscal policy effectiveness. Note also that, following the common practice, the CES bundle is normalised by the mass of varieties, $N$, to switch off the variety effect in the aggregate.

\(^3\) The use of a lump-sum tax is a common simplification in the literature, which reduces the distortionary role of the government. For further explanations see Molana and Moutos (1992), Heijdra and Van der Ploeg (1996) and Heijdra, et al. (1998) among others.
Each household is endowed with one unit of labour and an initial money holdings of \( \bar{M} \), and receives distributed profits \( \Pi \). In addition, it also supplies, inelastically, its unit of labour and at any point in time it can either be employed or be unemployed. When employed, a typical household works for a firm \( j \), supplying the effort level \( e_j > 0 \) and earning nominal wage \( W_j \). If unemployed, it receives from the government the nominal unemployment benefit \( B \) at no effort. Dropping the distinction between firms and setting profit income to zero (anticipating the symmetric equilibrium and elimination of profits through a free entry and exit process), a household’s budget constraint is

\[
P C + M = \begin{cases} W + \bar{M} - T, & \text{employed} \\ B + \bar{M} - T, & \text{unemployed} \end{cases}
\]

where \( M \) is the desired stock of money and \( C \) is the aggregate CES bundle of \( N \) individual varieties whose consumption is denoted by \( c_j \). The latter is determined according to the following demand

\[
c_j = \left( \frac{P_j}{P} \right)^{-\frac{1}{\gamma}} \left( \frac{C}{N} \right), \tag{5}
\]

where

\[
C = \left( \frac{1}{N} \int_{j \in N} c_j^{1-(1/s)} dj \right)^{1/(1-(1/s))},
\]

and \( P \) is the corresponding price index dual to \( C \); (5) maximises \( C \) above subject to the constraint \( \int_{j \in N} P_j c_j dj = PC \).
Household’s utility is given by

\[ V = v(C, M \div P) - \lambda \cdot f(e), \]  

(6)

where, in addition to the usual component \( v \) which we assume to be a Cobb-Douglas function,

\[ v(C, M \div P) = \frac{C^\alpha (M \div P)^{1-\alpha}}{\alpha(1-\alpha)^{1-\alpha}}, \]

the utility function also depends on the level of effort, \( e \), that an employed household will supply when working. The function \( f(e) \geq 0 \) captures the disutility of effort; \( \lambda = 1 \) for an employed household; and \( \lambda = 0 \) when the household is unemployed. Assuming that \( f(e) \) is taken as given (see Appendix A.1 for further details on the relevance of these and the derivation of the effort function) and maximising the utility function of an employed and an unemployed household subject to their respective budget constraint yields their consumption and money demand equations. Using \( L \) to denote the proportions of employed and invoking the normalisation

\[ L + u = 1, \]

(7)

the household sector’s aggregate consumption and money demand equations are

\[ C = \alpha \left( \frac{(1-u)W + uB + \bar{M} - T}{P} \right), \]

(8)

and

\[ \frac{M}{P} = (1-\alpha) \left( \frac{(1-u)W + uB + \bar{M} - T}{P} \right). \]

(9)

For simplicity, like most studies we assume complete separation between households’ and government’s consumption. Therefore, government consumption does not appear in households’ utility function. For some exceptions see, for example, Molana and Moutos (1989), Heijdra et al (1998) and Reinhorn (1998) who extend the original results by allowing for some substitution between the public and private consumption.
Given the above, the aggregate demand for the CES bundle, facing the monopolistically competitive firms, is \( Y = C + G \). On the assumption that each firm produces a distinct variety – given the incentive to specialise due to falling average costs explained low – the demand function facing firm \( j \) is \( y_j = c_j + g_j \) which is obtained by adding (5) and (2)\(^5\)

\[
y_j = \left( \frac{P_j}{P} \right)^s \left( \frac{Y}{N} \right). \tag{10}
\]

It is a straightforward exercise to show that \( P, Y \) and \( N \) satisfy the following

\[
Y = \left( N^{-1/s} \int_{j \in N} y_j^{1-(1/s)} \, dj \right)^{1/(1-1/s)} \tag{11},
\]

\[
\int_{j \in N} P_j y_j \, dj = PY, \tag{12}
\]

and

\[
P = \left( \frac{1}{N} \int_{j \in N} P_j \, dj \right)^{1/(1-r)} \tag{13}.
\]

Labour is assumed to be the only factor of production, and to be perfectly mobile between firms. Firm \( j \)'s technology is given by the following production function

\[
y_j = e_j L_j - \phi, \tag{14}
\]

where \( L_j \) is the variable labour input, \( e_j \) is labour productivity and \( \phi \) is a constant parameter reflecting the fixed cost of production assumed to be identical across firms. The increasing returns to scale, implied by falling average cost, therefore gives rise to the incentive for full specialisation from which a one-to-one correspondence between the mass of varieties and firms results.

\(^5\) We have followed the existing studies in assuming that \( G \) and \( C \) are similar CES bundles. See Startz (1989), Dixon and Lawler (1996), Heijdra and Van der Ploeg (1996) and Heijdra, et al. (1998) for further details.
We assume that $e_j$ is determined by workers’ attitude towards shirking and represents their optimal effort supply function, which depends on: i) the real value of the wage paid by the firm, $w_j = W_j / P$; ii) the real value of the unemployment benefit, $b = B / P$, which the government transfers to the unemployed household; and iii) the extent of unemployment in the economy captured by the unemployment rate $u$. Thus, we postulate the following effort supply function for a worker employed by firm $j$

$$e_j = e(w_j, b, u),$$

which is assumed to satisfy the following properties: (i) $e(w_j, b, u) \geq 0$ as $w_j \geq b$ and $e(w_j, b, u) = 0$ as $w_j = b$; (ii) $e'_w = \frac{\partial e_j}{\partial w_j} > 0$, $e'_b = \frac{\partial e_j}{\partial b} < 0$ and $e'_u = \frac{\partial e_j}{\partial u} > 0$; and (iii) to have plausible second and cross partial derivatives. In particular, we shall assume that

$$e''_{ww} = \frac{\partial ^2 e_j}{\partial w_j^2} < 0, \quad e''_{bw} = \frac{\partial ^2 e_j}{\partial b \partial w_j} < 0 \quad \text{and} \quad e''_{uw} = \frac{\partial ^2 e_j}{\partial u \partial w_j} > 0$$

(an example of this type of effort supply function, which satisfies the above properties and is obtained when workers maximise their expected utility from work, is explicitly derived in Appendix A1).

Each individual firm takes $P$, $Y$, $N$, $u$ and $B$ as given and chooses its ‘efficiency wage’ $W_j$ and its price $P_j$ so as to maximise its profit

$$\pi_j = P_j y_j - W_j L_j,$$

subject to the demand function in (10) and the production function in (14) as well as taking account of its workers’ reaction to the choice of $W_j$ which is given by the effort function in (15). The first order conditions are $\frac{\partial \pi_j}{\partial W_j} = 0$ and $\frac{\partial \pi_j}{\partial P_j} = 0$ whose solution imply the following wage and price setting rules.

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6 Given that the number of households is normalised to 1, $u$ is simply the proportion of unemployed households and is equivalent to the unemployment rate.

7 The second order conditions are satisfied as long as $s > 1$ and $e''_{uw} < 0$. 

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Equation (17) is a well-known result in the efficiency wage literature and implies that firms raise their wage rate up to the point where the effort function is unit elastic. Equation (18) is the usual mark-up pricing rule for a monopolistically competitive firm. In a symmetric equilibrium where all firms are identical, we drop the subscript \( j \) and write the above equations as

\[
e'_w(w, b, u) \cdot w = e, \tag{17'}
\]

and

\[
e(w, b, u) = \sigma \ w, \tag{18'}
\]

where \( \sigma = s/(s - 1) \). To see the (partial equilibrium) implications of these, first note that together they yield

\[
e'_w(w, b, u) = \sigma > 1. \tag{19}
\]

Next, totally differentiating (18') and taking account of (19) implies

\[
\frac{du}{db} = -\frac{e'_b}{e'_w} > 0, \tag{20}
\]

which shows that an increase in the benefit rate raises the unemployment rate. Finally, totally differentiating (17') and (18') and solving using (20) to eliminate \( db \) and \( dw \) yields (see Appendix A2)

\[
\frac{de}{du} = \left( \frac{\sigma}{e^*_{ww}} \right) \left[ \frac{e'_w e''_{bw} - e''_{bw}}{e'_b e_{ww} - e''_{bw}} \right]. \tag{21}
\]
Thus, under our assumptions regarding the shape of the effort function, (21) implies $de/du > 0$ which is consistent with the theoretical consensus that the net result of an increase in unemployment rate is to raise workers’ effort level.

We can use the above results to examine the way in which equilibrium output and unemployment are related to each other on the supply side. The symmetric equilibrium of the industry is obtained when entry eliminates profits,

$$\Pi = \int_{j \in N} \pi_j dj = PY - WL, \quad (22)$$

where $L = \int_{j \in N} L_j dj$ is total employment. Thus, through free entry and exit process $N$ adjusts to ensure $\Pi = 0$. Imposing this on (22) and solving for $Y$ gives $Y = wL$, from which upon substitution for $w$ from (18') we obtain

$$Y = \frac{1}{\sigma} eL. \quad (23)$$

Equation (23) may be interpreted as a ‘quasi-aggregate’ production function. It traces the combinations of aggregate employment and output $(L, Y)$ which satisfy the supply side equilibrium in which labour productivity is determined by an effort supply function and firms pay wages to induce workers to supply the effort level that maximises their profits. Or, put differently, these combinations give the equilibrium locus that describes how $Y$ changes as firms and workers respond to changes in $u$ while the wage is adjusted to ensure the resulting effort supply maximises profits. Totally differentiating (23) and noting that $dL = -du$ from (7), we obtain

$$\frac{dY}{du} = \left( \frac{e}{\sigma} \right) \left[ \left( \frac{1-u}{u} \right) \left( \frac{u de}{e du} \right) - 1 \right]. \quad (24)$$
Thus, provided that \( \frac{de}{du} \) in (21) is finite as \( u \to 1 \), we would expect the right-hand-side of (24) to be negative for sufficiently large levels of \( u \). Conversely, starting from sufficiently low levels of \( u \), we would expect the right-hand-side of (24) to be positive as long as \( \frac{de}{du} \) in (21) is positive, as explained above. Given these and assuming that \( \frac{de}{du} \) in (21) is continuous in \( u \), the equilibrium locus in \((u, Y)\) space may be depicted as in Figure 1.

**Figure 1. The Relationship between Output and Unemployment with Efficiency Wages**

The main implication of the model that we wish to stress is that it results in a change in \( dY/du \) from negative to positive as unemployment rate falls below a certain threshold, \( u = \bar{u} \). This is the rate of unemployment at which output attains its highest level, \( Y = \bar{Y} \). At such a point, the economy may be said to be operating without any slack despite supporting a level of ‘involuntary’ unemployment. Within the region where \( u > \bar{u} \), the economy exhibits the characteristics consistent with Okun’s Law since output and unemployment rate are negatively related. This situation corresponds to the ‘high-effort’ state where the economy can be said to be operating ‘efficiently’ and firms will have to employ more workers to meet a rise in aggregate demand. In contrast, the region where \( u < \bar{u} \) corresponds to the ‘low-effort’ state in which the economy may be said to be operating ‘inefficiently’. In this situation a higher level of output can be achieved at a lower level of employment since firms
will find it more profitable to meet the rise in demand by inducing the workers to raise their (optimal) effort supply. It is clear that in such an economy firms will (rationally) produce the same level of output, \( Y_0 \) say, employing either \((1 - u_e)\) inefficient workers or \((1 - u_s)\) efficient workers. Thus, multiple-equilibria can arise given the non-linear nature of the relationship between output and employment. As a result, the effect of a policy shock on employment and output depends on the initial equilibrium and unemployment can fall in the event of a positive shock only if the economy is operating in the ‘high-effort’ state. It can be easily shown that the (tax financed) fiscal multiplier is given by (see Appendix A3 for details),

\[
\frac{dY}{dG} = \frac{1}{1 - \frac{Y_D'}{Y_S'}}
\]

(25)

where \( Y_D' \) and \( Y_S' \) are the slopes of the aggregate demand and aggregate supply functions in \((P, Y)\) space, respectively. While \( Y_D' < 0 \) always holds, \( Y_S' \) can be positive or negative within the framework developed above\(^8\). Therefore, the effect of a fiscal expansion depends on the size and sign of the ratio of the slopes of the two functions. In particular, in the ‘high-effort’ state when \( Y_S' > 0 \) the multiplier will – as in the recent new Keynesian studies of the effect of fiscal policy – lie between zero and unity. But when the economy is operating in the ‘low-effort’ state and \( Y_S' < 0 \), the multiplier may either exceed unity or be negative (which resemble, respectively, the multipliers obtained under the typical Keynesian case and when more than full crowding out occurs). However, the former case, in which the rise in output will be accompanied by a fall in employment, corresponds to an unstable initial equilibrium.

\(^8\) It is a straightforward exercise to show that the aggregate supply function in \((P, Y)\) space is non-linear and can have more than one intersection with the aggregate demand. In such a situation, the equilibrium occurring where the aggregate demand curve is flatter than the (downward sloping) aggregate supply curve will be unstable (see Appendix A3 for further details on the slopes of aggregate demand and supply curves).
where \(-Y_s' > -Y_d' > 0\) whereas in the latter case \(-Y_d' > -Y_s' > 0\) and the initial equilibrium is stable.

3. Evidence
To empirically assess the implications of the model outlined in the previous section, regarding the relationship between output and unemployment rate, we have examined data on the level of output and the rate of unemployment from G7 countries – Canada, France, Germany, Italy, Japan, UK and US. More specifically, we have explored the strength of evidence to address the following questions:

(i) Does an ‘inversed U-shape’ specification adequately explain the way output is related to unemployment rate?

(ii) If so, then how does the ‘threshold rate of unemployment’ – the rate at which peak output occurs and which, according to our model, separates ‘low-effort’ from ‘high-effort’ states of production – vary over time?

(iii) How does the actual rate of unemployment compare with the ‘threshold rate of unemployment’ intertemporally?

To tackle this task, we have estimated a state space ‘local linear trend’ model using the Kalman-filter approach (see Harvey, 1989, for details), consisting of

\[
y_t = \begin{pmatrix} 1 & 0 & u_t \end{pmatrix} \begin{pmatrix} \alpha_t \\ \beta_t \\ \phi_t \end{pmatrix} + \xi_t, \quad \epsilon_t \sim i.i.d. \mathcal{N}(0, \sigma^2_{\epsilon})
\]

\[
\alpha_t = \alpha_{t-1} + \beta_{t-1} + \zeta_t, \quad \zeta_t \sim i.i.d. \mathcal{N}(0, \sigma^2_{\zeta}),
\]

\[
\beta_t = \theta \beta_{t-1} + \xi_t, \quad \xi_t \sim i.i.d. \mathcal{N}(0, \sigma^2_{\xi}), \quad 0 < \theta < 1.
\]
\[
\phi_t = \phi_{t-1} + \eta_t; \quad \eta_t \sim i.i.d. \left(0, \sigma^2_\eta \right),
\]  \hspace{1cm} (29)

and

\[
\delta_t = \delta_{t-1} + \psi_t; \quad \psi_t \sim i.i.d. \left(0, \sigma^2_\psi \right).
\]  \hspace{1cm} (30)

To capture the main feature of the non-linearity implied by the model, the measurement equation (for output, \(y\)) in (26) is assumed to be a quadratic function\(^9\) (of the unemployment rate, \(u\)) subject to an additive random shock, \(\epsilon\), and with randomly evolving (state) parameters \((\alpha, \beta, \phi, \delta)\). The shock \(\epsilon\) is assumed to be drawn independently from a normal distribution and the evolution process over time postulated for the state vector \((\alpha_t, \beta_t, \phi_t, \delta_t)\) is described by the transition equations (27)-(30). The generality allowed by this set up is particularly useful when it is applied to bivariate relationships which both: (a) involve variables that have strong secular pattern and/or are subject to cyclical fluctuations\(^{10}\); and (b) are, by construction, restricted and fail to condition explicitly on a host of other potentially relevant variables\(^{11}\). The state-space representations, in this context, are very flexible since the non-stationary processes generating \(\phi\) and \(\delta\) are allowed to evolve in a manner capable of capturing any fundamental changes, which may have occurred in the historical relationship between \(y\) and \(u\). Moreover, to account for trends in output growth and the unemployment rate over our estimation period (1960-2001), we have allowed for local linear trends where both the level, \(\alpha_{t-1}\) and the slope, \(\beta_{t-1}\) vary over time (see Harvey, 9

\(^9\) While there are a wide variety of alternative non-linear functions capable of capturing the non-monotonic link between \(y\) and \(u\) predicted by our theory, we have opted for the simplest and most parsimonious of these.

\(^{10}\) Both output and unemployment have these properties and the estimation method adopted here is a superior alternative to isolating the secular and cyclical components by filtering the series before checking how they relate to each other over time.

\(^{11}\) In the absence of any explicit dynamics, we employ contemporaneous values of both output and the unemployment rate. This approach might reasonably be expected to yield biased parameter estimates, given the joint endogeneity of the variables. To assess the extent of this bias we also experimented with IV and GMM estimation and found any biases to be quantitatively negligible. To preserve space, these latter results are not reported here but will be made available on request.
To estimate (26) allowing for (27)-(30), we require starting values for the state vector and its variance-covariance matrix, \((\alpha_0, \beta_0, \phi_0, \delta_0)\) and \(P_0\). In the absence of any prior information on the initial distribution\(^\text{13}\), we have employed a diffuse prior which involves setting the starting values of the coefficients equal to zero and letting \(P_0 = I \lambda\), where \(I\) is the conformable unit matrix and \(\lambda\) is a very large number (see Harvey, 1989, pg. 121).

Empirical support for our theory, within the context of the questions (i) and (ii) posed above, at the beginning of this section, requires that:

(i)\(^\prime\) \(\phi_t\) must be significantly greater than zero, \(\delta_t\) must be significantly less than zero, and the estimated residuals, \(\hat{\epsilon}_t\), must be stationary and unpredictable;

(ii)\(^\prime\) The ‘threshold rate of unemployment’, denoted by \(\bar{u}_t\) and given by 
\[
\bar{u}_t = \phi_t / (-2\delta_t)
\]
from the quadratic function in (26), should be significantly greater than zero.

To examine these, we obtained filtered estimates of the state vector for each of the G7 countries. Data are quarterly over the period 1960:Q1-2001:Q1 except the French data which do not start until 1964:Q4 and the results are reported in Table 1 below. Columns (I) and (III) give, for \(t = T\), the filtered estimates of \(\phi_t\) and \(\delta_t\), and column (V) gives the corresponding estimate for the threshold rate of unemployment, 
\[
\bar{u}_t = \phi_t / (-2\delta_t)
\] . Columns (II), (IV) and (VI) report, respectively, the proportions of estimates within the sample period for which the following null hypotheses cannot be rejected at the 5% critical level: \(\phi_t > 0\); \(\delta_t < 0\); \(\bar{u}_t > 0\). Table 2 reports estimates of autocorrelation and partial

\(^{12}\) Note that in contrast to the other parameters which follow random walks, \(\beta_t\) is assumed to follow a stationary AR(1) process. This assumption is employed since a non-stationary process for this parameter would imply \(y_t \sim I(2)\). This, however, is against the widely acknowledged stylised fact that the growth rate of output is stationary, which is also supported by our data set. For example, univariate evidence based on ADF, weighted-symmetric and Phillips-Perron tests suggest that \(y_t\) has only one unit root (this evidence is not presented here but will be made available on request).

\(^{13}\) Given that three of the four transition equations are non-stationary, the unconditional distribution of the state vector is not defined.
autocorrelation coefficients, denoted by AC and PAC respectively, for 8 lags for the estimated residuals. These results suggest that the quadratic specification adequately captures the non-linear shape of the relationship between output and unemployment rate predicted by our model, beyond any co- or counter-movements due to secular and/or cyclical patterns in the underlying series. Recall that this type of non-linear specification implies that at any point in time the economy can be in one of the following three possible states: $u_t < \bar{u}_t$, $u_t = \bar{u}_t$ and $u_t > \bar{u}_t$. However, as we have pointed out, the standard explanations – such as Okun’s Law which predict a trade off between output and unemployment – would imply a monotonic negative relationship between $y_t$ and $u_t$ hence requiring, within our framework, that the actual rate of unemployment is always above the threshold rate, i.e. $u_t > \bar{u}_t$. Such explanations therefore cannot account for the significant occurrence of $u_t < \bar{u}_t$ or $u_t = \bar{u}_t$.

To see how $u_t$ compares with $\bar{u}_t$, in Figure 2 below we illustrate the frequency of the occurrence of each of the three states for each country. The graphs show the ratio of the actual unemployment rate to the estimated threshold rate for each period, i.e. $u_t / \hat{\bar{u}}_t$, and the table in the bottom-right corner gives the percentage of significant occurrence of each state at 5% and 10% critical level. These results show that whilst the evidence that $u_t < \bar{u}_t$ has occurred significantly at some periods cannot be ruled out, periods in which $u_t = \bar{u}_t$ seem to dominate and only German data shows a strong support for $u_t > \bar{u}_t$.

Finally, we note that our evidence is in line with the findings reported by studies that have examined the behaviour of labour productivity in the industrialised countries and provide evidence on the way in which labour productivity has changed over the last few decades. Recent examples include Disney, et al. (2000), Barnes and Haskel (2000), Marini and Scaramozzino (2000), Bart van Ark et al. (2000) and Sala-i-Martin (1996). However, the
evidence provided in these studies is usually interpreted using one of the micro-theory based explanations underlying the behaviour of labour productivity. These may, in general, be divided into two categories. The first concentrates on the productivity gains that can be realised through: i) improved skill due to training; ii) increased efficiency due to progress in management and restructuring; and iii) rising physical productivity of other factors of production due to R&D, etc.. The second category emphasises market forces and sees competition and market selection as the main motivation behind the rise in efficiency. The separating line between these two accounts is not very clear in the sense that the second will have to be achieved through the first when the economy is operating efficiently. However, if the economy happens to be in an inefficient phase, market forces can act directly without having to induce any of the factors in the first category. The efficiency wage hypothesis is a typical example of this case.

4. Summary and Conclusions
The main motivating factor underlying our study has been the possibility that a positive policy shock might give rise to adverse employment effects. This result is unlikely if Okun’s Law holds and output and unemployment rate are always related positively, once we take account of secular and cyclical changes. We outline a simple theoretical setting which can account for deviations from Okun’s Law. We show that, if at high unemployment rates firms can induce workers to supply more effort, the equilibrium relationship between aggregate output and unemployment rate can be positive provided the gain in productivity is sufficiently large to outweigh the negative effect of the reduction in employment. Our evidence, based on data from G7 countries over the period 1960-2001, shows strong support for deviation from Okun’s Law. Using an estimation method which allows for trends,
cyclical changes and breaks, we find that only German data strongly favour a persistent negative relationship between the level of output and rate of unemployment.

Clearly, our results – which complement those of the literature on the effects of contractionary fiscal policy (see, for details, Barry and Devereux, 1995) and on the positive effects of unemployment insurance (see Acemoglu and Shimer, 2000) – suggest that plausible circumstances do exist in which market imperfections pose serious obstacles to the smooth working of expansionary and/or stabilization policies. We show that the economy can rationally operate at an inefficient equilibrium, and that positive demand shocks in such circumstances will have perverse effects. Accordingly, we conclude by stressing Lindbeck’s (1992) concerns about the effectiveness of macroeconomic stabilisation policy in the presence of labour and product market imperfection, which are echoed by our results.

Finally, given our definition of the threshold rate of unemployment and the evidence that in a number of countries the actual unemployment rate has a tendency to coincide with a time varying estimate of such a threshold rate, exploring the links between the latter and the time-varying NAIRU (see Gordon, 1997b for details) can throw light on the determination of the natural rate of output and hence provides an interesting direction for future research.
5. References


Manning, A. (1992), Productivity growth, wage setting and the equilibrium rate of unemployment, Centre for Economic Performance, Discussion paper No 63.

Marini G. and P. Scaramozzino (2000), Endogenous growth and social security, CeFiMS DP 02/00/02, Centre for Financial and Management Studies, SOAS, University of London


6. Appendix

A1. Derivation of the Effort Supply Function $e(w, u, b)$

This appendix explains how a specific effort supply function such as that in equation (15) can be derived within the framework of the efficiency wage hypothesis where, following common practice, the agent is assumed to maximise the expected utility of remaining in employment.

We assume that all households participate in the labour market and at any point in time a household can be in one of the following states: (i) employed (working); (ii) being fired (when caught shirking at work); (iii) unemployed (being without a job); or (iv) being hired (finding a job). Let the utility indices corresponding to of the above states be denoted as follows:

(i) employed (working): $V^E$
(ii) being fired (losing one’s job): $V^F$
(iii) unemployed (being without a job): $V^U$
(iv) being hired (finding a job): $V^H$

$V^U$ and $V^E$ can be obtained as follows. Disregarding the money holdings and taxes (which are the same for all states) and focusing on unemployment benefit or wage as the only source of work-related income, the utility function in equation (6) implies that the indirect utility of an unemployed and an employed household is, respectively,

$$V^U = B / P,$$  \hspace{1cm} (A1.1)

and

$$V^E = (W / P) - f(e).$$ \hspace{1cm} (A1.2)

While (A1.1) is straightforward, (A1.2) needs some explanation regarding $f(e)$. We shall assume $f’ > 0$ and $f’’ \geq 0$ which implies that the disutility of effort rises with a non-decreasing rate. In particular, we shall use the explicit form $f(e) = ke^z$ where $k > 0$ is a scaling factor.
Finally, we need to specify $V^H$, which is the satisfaction a household attaches to finding a job or being hired. But the utility associated with this state is in principle not distinguishable from $V^E$ and for simplicity we let

$$V^H = V^E,$$  \hspace{1cm} (A1.3)

The probabilities associated with moving from one state to another are assumed to be determined as follows:

(a) **Probability associated with being fired when shirking, $F$.**

We assume that shirking is the only reason for being fired (we do not explicitly model the monitoring technology). Therefore, *ceteris paribus*, $F$ is a monotonic function of the effort level, $e$. Thus,

$$F = F(e); F(0) = 1; F(1) = 0; \frac{dF}{de} < 0.$$  

For simplicity, normalise the maximum possible effort to unity and let

$$F = 1 - e.$$  \hspace{1cm} (A1.4)

(b) **Probability associated with finding a job, or being hired, when unemployed, $H$.**

We assume that the labour force is homogeneous and, *ceteris paribus*, $H$ is a monotonic function of the unemployment rate, $u$ (we do not explicitly model the search technology). Thus,

$$H = H(u); H(0) \leq 1; H(1) = 0; \frac{dH}{du} < 0.$$  

For simplicity we let

$$H = 1 - u.$$  \hspace{1cm} (A1.5)

We define the optimal level of effort as that which maximises a household’s expected utility of remaining in employment. The latter is denoted by $R(e)$ and is, by definition, given by
\[ R(e) = (1 - F)V^E + FV^F. \]  

(A1.6)

Also, given that a ‘fired’ worker can either be hired or remain unemployed, we let \( V^F \) be a weighted average of \( V^H \) and \( V^U \). Thus,

\[ V^F = HV^H + (1 - H)V^U. \]  

(A1.7)

Equations (A1)-(A7) yield

\[ R(e) = e(w - ke^2) + (1 - e)((1 - u)(w - ke^2) + ub), \]

where \( w=W/P \) and \( b=B/P \). This equation can be rearranged as

\[ R(e) = -uke^3 - (1 - u)ke^2 + u(w - b)e + ((1 - u)w + ub). \]  

(A1.8)

The agent takes \((w, b, u)\) as given and chooses \( e \) to maximise \( R(e) \). The first order condition for this is \(-e^2(2/3)(1-u)/ue + (1/3k)(w-b) = 0\). This has two roots of which only one is positive, which also satisfies the second order for a maximum and can, after some normalisation, be written as

\[ e = \left[ \gamma (w-b) + \left( \frac{1-u}{u} \right)^2 \right]^{1/2} \frac{1-u}{u}, \]  

(A1.9)

where \( \gamma = 3k \). It is clear that equation (A1.9) satisfies our specified conditions, since

\[ e(w, b, u) \geq 0 \quad \text{as} \quad w \geq b; \quad e(w, b, u) = 0 \quad \forall u \in (0, 1) \quad \text{as} \quad w = b; \quad e' = \frac{\partial e}{\partial w} > 0; \]

\[ e'_b = \frac{\partial e}{\partial b} < 0; \quad e'_u = \frac{\partial e}{\partial u} > 0; \quad e''_{ww} = \frac{\partial^2 e}{\partial w^2} < 0; \quad e''_{bw} = \frac{\partial^2 e}{\partial b \partial w} > 0; \quad \text{and} \quad e''_{uw} = \frac{\partial^2 e}{\partial u \partial w} > 0. \]

(A2.1)

A2. Derivation of Equations (20) and (21).

We use the following equations (15), (17´), (18´) and (19) which are reproduced below as (A2.1)-(A2.4), respectively,

\[ e = e(w, b, u), \]  

(A2.1)
\[ e'_w(w, b, u) \cdot w = e, \quad (A2.2) \]
\[ e(w, b, u) = \sigma \cdot w, \quad (A2.3) \]
\[ e'_w(w, b, u) = \sigma. \quad (A2.4) \]

First, totally differentiating (A2.3) yields
\[ e'_w dw + e'_b db + e'_u du = \sigma dw, \quad (A2.5) \]
and substituting from (A2.4), i.e. \( e'_w = \sigma \), in (A2.5) we obtain
\[ e'_b db + e'_u du = 0, \quad (A2.6) \]
which is solved to yield equation (20).

Next, totally differentiating (A2.4) implies
\[ e''_w dw + e''_b db + e''_u du = 0, \quad (A2.7) \]
and using (A2.6) to eliminate \( db \) we have
\[ e''_w dw + e''_b \left( -\frac{e'_u}{e'_b} \right) du + e''_u du = 0, \quad (A2.8) \]
which can be solved for \( dw \) to give
\[ dw = \left( \frac{1}{e''_w} \right) \left[ e''_b \left( \frac{e'_u}{e'_b} \right) - e''_u \right] du, \quad (A2.9) \]
Substituting from (A2.9) into \( de = \sigma dw \) implied by (A2.3), we obtain equation (21).

We derive the fiscal multiplier as follows. First, the aggregate demand function (AD) is derived by noting that \( Y = C + G \), where \( C \) is obtained by solving equations (8) and (9), i.e.
\[ C = \left[ \alpha \ / (1 - \alpha) \right] \left( \frac{\bar{M}}{P} \right). \]
Hence,
\[ Y = G + \left( \frac{\alpha}{1 - \alpha} \right) \left( \frac{\bar{M}}{P} \right). \quad (A3.1) \]
Totally differentiating (A3.1), for any give $M$, then implies that on $AD$,

$$dY = dG - \left( \frac{\alpha M}{1 - \alpha P^2} \right) dP.$$ \hspace{1cm} (A3.2)

Next, recalling from equations (23) and (7) that $Y = wL$ and $L = 1 - u$, we obtain, on the aggregate supply $(AS)$ side, $Y = w(1 - u)$, which can be totally differentiated to yield

$$dY = (1 - u)dw - wdu,$$ \hspace{1cm} (A3.3)

which upon substitution from (A2.9) can be written as,

$$dY = \left\{ \left( \frac{1 - u}{e_{aw}^'} \right) \left( \frac{e_{aw}^' - e_{aw}^''}{e_{bw}^'} \right) - w \right\} du.$$ \hspace{1cm} (A3.4)

Also, using (A2.6) and the fact that for any given $B$, $db = -\frac{B}{P^2} dP$, we obtain

$$du = \left( \frac{e_{aw}^'}{e_{aw}^''} \right) \left( \frac{B}{P^2} \right) dP,$$ \hspace{1cm} (A3.5)

which can be substituted in (A3.4) to give the reaction of output to a change in the price level on the aggregate supply $(AS)$,

$$dY = \left\{ \left( \frac{1 - u}{e_{aw}^'} \right) \left( \frac{e_{aw}^' - e_{aw}^''}{e_{bw}^'} \right) - w \right\} \left( \frac{e_{aw}^'}{e_{aw}^''} \right) \left( \frac{B}{P^2} \right) dP.$$ \hspace{1cm} (A3.6)

Simplifying notation and writing (A3.2) and (A3.6) as

$$dY = Y'_D dP + dG$$

$$dY = Y'_S dP$$

where $Y'_D$ and $Y'_S$ are the slopes of $AD$ and $AS$ in $(P, Y)$ space. Solving the above to eliminate $dP$ we obtain the fiscal multiplier in equation (25), namely,

$$\frac{dY}{dG} = \frac{1}{1 - \frac{Y'_D}{Y'_S}}.$$ \hspace{1cm} (A3.5)
Table 1. Selected Results from Estimation of Equation (25) based on G7 data for 1964(1)-2001(1)

<table>
<thead>
<tr>
<th></th>
<th>(I) $\hat{\phi}_T$</th>
<th>(II) T $\hat{\delta}_T$</th>
<th>(III) % over sample for which $\hat{\delta}_T &gt; 0$ is not rejected at 5%.</th>
<th>(IV) $\hat{\delta}_T$</th>
<th>(V) $\hat{\delta}_T$</th>
<th>(VI) % over sample for which $\hat{\delta}_T &gt; 0$ is not rejected at 5%.</th>
</tr>
</thead>
<tbody>
<tr>
<td>US</td>
<td>2.49 (0.232)</td>
<td>100%</td>
<td>-0.250 (0.036)</td>
<td>100%</td>
<td>5.62 [1.21]</td>
<td>100%</td>
</tr>
<tr>
<td>Canada</td>
<td>2.26 (0.223)</td>
<td>100%</td>
<td>-0.124 (0.022)</td>
<td>100%</td>
<td>9.14 [2.26]</td>
<td>92%</td>
</tr>
<tr>
<td>UK</td>
<td>2.47 (0.640)</td>
<td>100%</td>
<td>-0.158* (0.178)</td>
<td>100%</td>
<td>8.02* [5.03]</td>
<td>95%</td>
</tr>
<tr>
<td>France</td>
<td>4.73 (0.776)</td>
<td>100%</td>
<td>-0.258 (0.124)</td>
<td>100%</td>
<td>9.13 [2.13]</td>
<td>80%</td>
</tr>
<tr>
<td>Germany</td>
<td>21.06 (4.60)</td>
<td>100%</td>
<td>-2.47 (0.593)</td>
<td>100%</td>
<td>4.26 [0.07]</td>
<td>100%</td>
</tr>
<tr>
<td>Italy</td>
<td>4.75 (0.560)</td>
<td>100%</td>
<td>-0.224 (0.082)</td>
<td>100%</td>
<td>10.58 [2.77]</td>
<td>98%</td>
</tr>
<tr>
<td>Japan</td>
<td>4.26 (1.04)</td>
<td>100%</td>
<td>-0.585* (0.231)</td>
<td>96%</td>
<td>3.65 [0.59]</td>
<td>85%</td>
</tr>
</tbody>
</table>

Notes:
(a) The initial 4 years (16 observations) were used to allow the filtered estimates sufficient time to stabilise and were excluded in obtaining estimates in this table.
(b) The statistical significances of $\hat{\phi}_T$ and $\hat{\delta}_T$ are based on their asymptotic standard errors. The numbers in parenthesis are the asymptotic standard errors for the final state vector, $t = T$.
(c) To assess the statistical significance of $\hat{\delta}_T$ on a period-by-period basis we have conducted a parametric bootstrap using 2000 replications for each quarter. The numbers is square brackets are the bootstrapped standard errors for the final period.
(d) An asterisk indicates not significant at the 5% level.
(e) The local linear trend components were not significant for German data and hence were excluded in final estimation for that country.

Table 2: Autocorrelation and Partial Autocorrelation Coefficients for the Estimated Residuals of Equation (25)

<table>
<thead>
<tr>
<th></th>
<th>AC</th>
<th>PAC</th>
<th>AC</th>
<th>PAC</th>
<th>AC</th>
<th>PAC</th>
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<th>PAC</th>
<th>AC</th>
<th>PAC</th>
</tr>
</thead>
<tbody>
<tr>
<td>US</td>
<td>0.596</td>
<td>-0.077</td>
<td>0.305</td>
<td>-0.066</td>
<td>0.107</td>
<td>-0.217</td>
<td>-0.12</td>
<td>-0.141</td>
<td>-0.216</td>
<td>-0.048</td>
<td>0.105</td>
<td>-0.152</td>
<td>-0.173</td>
<td>-0.173</td>
</tr>
<tr>
<td>Canada</td>
<td>0.809</td>
<td>-0.19</td>
<td>0.59</td>
<td>-0.111</td>
<td>0.377</td>
<td>-0.142</td>
<td>0.17</td>
<td>0.164</td>
<td>0.073</td>
<td>-0.04</td>
<td>0.009</td>
<td>-0.084</td>
<td>0.003</td>
<td>-0.021</td>
</tr>
<tr>
<td>UK</td>
<td>0.274</td>
<td>-0.194</td>
<td>-0.104</td>
<td>-0.09</td>
<td>0.112</td>
<td>-0.009</td>
<td>0.17</td>
<td>0.112</td>
<td>-0.021</td>
<td>-0.042</td>
<td>-0.117</td>
<td>-0.006</td>
<td>0.114</td>
<td>-0.09</td>
</tr>
<tr>
<td>France</td>
<td>0.653</td>
<td>-0.127</td>
<td>0.353</td>
<td>0.088</td>
<td>0.239</td>
<td>0.09</td>
<td>0.09</td>
<td>0.036</td>
<td>0.11</td>
<td>0.09</td>
<td>0.09</td>
<td>0.112</td>
<td>0.09</td>
<td>0.09</td>
</tr>
<tr>
<td>Germany</td>
<td>0.599</td>
<td>0.105</td>
<td>-0.066</td>
<td>0.085</td>
<td>-0.005</td>
<td>0.036</td>
<td>-0.015</td>
<td>0.083</td>
<td>0.129</td>
<td>-0.074</td>
<td>0.084</td>
<td>0.114</td>
<td>-0.09</td>
<td>0.09</td>
</tr>
<tr>
<td>Italy</td>
<td>0.147</td>
<td>0.26</td>
<td>0.276</td>
<td>0.065</td>
<td>0.128</td>
<td>-0.031</td>
<td>0.103</td>
<td>0.13</td>
<td>0.172</td>
<td>0.042</td>
<td>0.196</td>
<td>0.052</td>
<td>0.025</td>
<td>0.025</td>
</tr>
<tr>
<td>Japan</td>
<td>0.001</td>
<td>-0.044</td>
<td>0.105</td>
<td>-0.091</td>
<td>0.038</td>
<td>-0.022</td>
<td>-0.013</td>
<td>0.076</td>
<td>-0.127</td>
<td>0.196</td>
<td>-0.088</td>
<td>-0.106</td>
<td>0.052</td>
<td>0.025</td>
</tr>
</tbody>
</table>
Figure 2: Relationship between Actual and Threshold Unemployment Rates

US: Actual to Threshold Unemployment Rates

France: Actual to Threshold Unemployment Rates

Italy: Actual to Threshold Unemployment Rates

Canada: Actual to Threshold Unemployment Rates

Germany: Actual to Threshold Unemployment Rates

Japan: Actual to Threshold Unemployment Rates
proportion of $u_i < \bar{u}_i$
proportion of $u_i = \bar{u}_i$
proportion of $u_i > \bar{u}_i$

sig. at 5%
sig. at 10%
sig. at 5%
sig. at 10%
sig. at 5%
sig. at 10%

US
Canada
UK
France
Germany
Italy
Japan

0.09
0.05
0.01
0.00
0.19
0.00
0.00
0.20
0.09
0.03
0.00
0.19
0.00
0.01
0.91
0.95
0.77
0.94
0.01
1.00
1.00
0.77