Cumulative Causation, the Welfare State and International Specialisation

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and

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Abstract
We model a small open economy which produces a high-tech and a low-tech good and whose government pursues redistributive policies financed through distortionary taxation. With vertical linkages between sectors and with unionised labour markets, we analyse the effects of welfare state provision on: (1) the pattern of the country’s trade, (2) the depth of the division of labour within the economy, and (3) the country’s welfare. We show that an increase in the size of the welfare state might have positive effects on the country’s income and on the extent to which the country specialises in the high-tech sector.

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1. INTRODUCTION

This paper focuses on the effects of welfare state provision on a country’s economic performance.

The last fifty years have witnessed a world-wide increase in the use of fiscal redistribution. Indeed, albeit to different extents, most countries’ ascent on the industrialisation ladder has indisputably been accompanied by an increase in the relative size of their welfare state programmes. However, in the 1980s and 1990s a new conventional wisdom of ‘good governance’ has emerged which questions governments’ role in the economy. One facet of this conventional wisdom is that ‘rolling-back the welfare state’ is an inevitable consequence of ‘globalisation’, the latter being perceived as eroding the power of governments to independently shape their economies. ‘Successful competition’ in international markets is thus undermined by welfare state policies that introduce rigidities in labour markets and have distortionary effects through the taxation required to finance them. A corollary of this view is that labour market flexibility and reduced union power are essential to ‘successfully competing’ in the global economy. Interestingly, this conventional wisdom partly conflicts with some of the stylised facts emerging from the existing empirical evidence that indicate a positive correlation between openness and the scope of government (e.g. Cameron, 1978; Rodrik, 1996) and between openness, union size and degree of wage setting centralisation (e.g. Cameron, 1978; Agell, 1999).

Unfortunately, despite the attention they have attracted in policy debates, the academic literature has hardly dealt with the relationship between economic integration and the welfare state\(^1\). In the theoretical literature that has recently started to emerge, three main approaches can be identified. The first focuses on the distortionary effects of public expenditure and taxation. Alesina and Perotti (1997) find that, in the presence of labour unions, welfare expenditure financed through labour income taxation has negative effects on competitiveness – measured by relative unit labour cost – because unions transfer the burden of taxation on to firms through higher wages. A second argument rests on the effects of openness on a country’s exposure to risk. On the one hand, by increasing the potential for risk diversification (particularly in financial assets) openness reduces the overall exposure to risk. On the other hand, because of a higher specialisation of production, international integration will increase the exposure to foreign shocks. Whilst Wildasin (1995), within a static partial equilibrium

\(^1\) For a general discussion of these issues see Atkinson (1997).
model, finds that international integration reduces the need for social insurance. Rodrik (1997, 1998) argues that, by increasing volatility, openness enhances the need for social insurance. In support of this conclusion Rodrik presents empirical evidence that more open economies have larger public sectors which act as risk insurers. Andersen (2000) develops an inter-temporal general equilibrium two-country model of international trade that reconciles the two arguments above. On the one hand the model supports the view that integration increases the distortionary effects of public sector activities, thus calling for a reduction of the welfare state. On the other, it shows how higher degrees of integration may increase volatility thus requiring a rise in the provision of social insurance. The third approach is developed within the closed-economy endogenous growth framework and focuses on the effects of social transfers on the level of economic efficiency. Sala-i-Martin (1996) argues that transfers can have a positive effect on the average stock of human capital, effectively by buying the elderly out of the workforce, thus leading to increases in output per capita. Marini and Scaramozzino (2000) show how the introduction of an unfunded balanced-budget pension scheme can lead to permanently higher capital and output in the presence of learning-by-doing dynamics stemming from capital accumulation.

We intend to contribute to the assessment of these issues by highlighting a different mechanism through which welfare state policies interact with openness in affecting a country’s economic performance. The central idea of this paper is that social security programmes may lead to higher levels of economic efficiency by improving the exploitation of potential aggregate economies of scale. The importance of the latter has been formalised by the endogenous growth literature and the fact that positive externalities may arise for the whole economy from producing in certain sectors has been informing government policies around the world. Despite this, the theoretical underpinnings of the typical arguments levied against the welfare state do not allow for the effects of these externalities. However, any meaningful debate about the sustainability of welfare state programmes has to be based on analytical frameworks that recognise the importance of these externalities because they affect – in a rather complex and therefore not easily predictable fashions – the way in which economic policy impacts on the economy’s performance. We shall therefore set up a framework within

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2 In the UK, the importance of specialisation in knowledge based (fast growing demand) activities (DTI, 1998) capable of generating externalities for the economy as a whole is a cornerstone of the current policy framework.
which the role of economy-wide externalities in determining the relationship between welfare state and competitiveness can be captured.

Although its precise definition, nature, extent and consequences still remain a matter of contention, globalisation is typically meant to refer to the process through which national economies become more open, and is implicitly purported as reducing the economic size of countries and their monopoly power in world markets\(^3\). Hence, a small open economy assumption seems to be a plausible and analytically valuable starting point to analyse the effects of welfare state policies on economic performance. Of the many dimensions of ‘globalisation’, we shall concentrate on free-trade in final goods markets and rule out factor mobility\(^4\). Furthermore, given that the social and political costs of rolling back the welfare state are likely to be more significant in industrialised countries where welfare states are larger, we choose to focus the analysis on an industrial economy. Therefore, the model set-up will be based on a number of assumptions meant to characterise such an economy. First, the country’s government pursues redistributive policies financed through distortionary taxation. Second, labour markets are unionised with unemployment emerging in equilibrium. With unionisation, income redistribution policies financed by labour income taxes will affect firms’ costs since the unions will transfer part of the burden of taxation to employers via higher wages. Given that the degree of distortion generated by unionisation is known to depend on the degree of coordination of unions’ decisions (e.g. Calmfors and Driffill, 1988; Summer, Gruber and Vergara, 1993; Rama, 1994), we allow for different degrees of wage setting centralisation. The final crucial feature of the model concerns the source of the aggregate economies of scale. It is now widely accepted that a typical implication of industrial development is the increasing ‘indirectness’ of production processes, with final goods sectors relying more and more on highly specialised intermediate inputs. We shall therefore assume an input-output structure with vertical linkages between an intermediate and two final goods (one high-tech and one

\(^3\) According to Alesina and Spolaore (1997), the physical size of countries is also to an extent endogenous to the process of economic integration. The latter, by reducing the constraints to market size posed by a country’s political boundaries, may lead to political fragmentation thus allowing countries to reduce the cost of cultural and ethnic heterogeneity.

\(^4\) To a great extent, capital mobility is at the core of the debate about the compatibility of welfare states with international integration. It should, however, be noted that social security activities are primarily financed by income taxes and do not rest significantly on capital taxation (e.g. in the EU on average capital taxation only accounts for 7.5% of public sector revenue). Hence, the argument that capital mobility, by shrinking the tax base, poses further threats to the welfare state may not be so crucial. Allowing for international capital mobility does not substantially alter the main results of this paper (see Molana and Montagna, 2000).
low-tech) sectors\(^5\). The increasing returns to scale stemming from this input-output structure give rise to a *circularity* between range of intermediates, economy-wide efficiency and pattern of specialisation in production and trade. Thus, to the extent that government policies affect market structure, they will also impact on the availability of intermediates, on their aggregate productivity and on the economy’s trade performance.

Within this framework, we shall examine the effects of social insurance policies (in the form of unemployment benefits) and of different labour market institutional settings on a country’s economic performance. The latter is defined as a country’s ability to maintain or expand its present level of income, to deepen its division of labour and improve its degree of specialisation in high-tech sectors.

In general, our findings suggest that in a price-taking economy characterised by complementarities in production, an increase in the size of the welfare state can have positive effects on the country’s real income and on the extent to which it specialises in the high-tech sector. As in Alesina and Perotti (1997), larger welfare states do lead to higher distortions, particularly at intermediate levels of wage setting centralisation (when unions are strong enough to reap substantial rents but not large enough to take account of the links between their actions and the government budget constraint). Contrary to Alesina and Perotti, however, our results point to the fact that larger welfare states do not have *unambiguously negative* effects on a country’s ‘competitiveness’ and that the ‘disruption’ of higher wages may instead positively affect the overall performance of an economy. This is because, in the presence of vertical linkages between sectors, higher wages in the intermediate sector could result in a higher demand for intermediates. In these circumstances, those factors responsible for a higher wage, such as a more generous welfare state and/or an increase in unions’ monopoly power, will lead to a deepening in the division of labour and to an increase in the degree of specialisation in the high-tech sector.

The rest of the paper is organised as follows. Section 2 outlines the model, Section 3 derives the general equilibrium, Section 4 carries out the policy analysis and Section 5 draws some conclusions.

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\(^5\) The existing empirical evidence reveals important inter-industry connections leading to external returns to scale in manufacturing (e.g. Caballero and Lyons, 1992; Bartelsman, Caballero and Lyons, 1994). At a theoretical level, the importance of vertical linkages as a major source of economy-wide increasing returns to scale has been widely acknowledged. See among others: Eitner (1982), Matzuyama (1995), Okuno-Fujiwara (1988), Rodriguez-Clare (1996), Rodrik (1996), Venables (1996).
2. THE MODEL

We consider a small open economy consisting of an upstream intermediate goods sector and of two downstream final good industries. The downstream industries produce two final goods \( (Y_1 \text{ and } Y_2) \) which are homogeneous and are freely traded in world markets. The upstream industry produces an intermediate good \( (X) \) that comes in a continuum of varieties. This good can be thought of as consisting of highly specialised producer services and other intangible inputs such as knowledge. As all other production factors, these intermediate varieties are assumed to be non-traded\(^6\).

2.1. Consumers

The representative individual has the homothetic utility function

\[
U = \frac{Y_1^\mu Y_2^{1-\mu}}{(1-\mu)^{1-\mu} \mu^{-\mu}} + (1-\xi) \tilde{V},
\]

which can be taken to represent the aggregate preferences of the consumer sector. In (1), \( Y_h \) \((h=1,2)\) are consumption of the final goods. The representative individual is assumed to be endowed with one unit of labour, supplied inelastically. The individual is employed if \( \xi = 1 \) and unemployed if \( \xi = 0 \). \( \tilde{V} \) is the utility of leisure. Constrained optimisation of (1) yields the demand functions

\[
Y_1^d = \mu \frac{M}{P_1},
\]

\[
Y_2^d = (1-\mu) \frac{M}{P_2},
\]

\(^6\) The non-tradability of intermediates is frequently assumed in the literature to capture the importance of proximity of the intermediate sector to final good industries (e.g. Rodriguez-Clare, 1996, and Rodrik, 1996). As we argue in the conclusions, however, a natural extension of this paper will be to relax this assumption, given that the empirical evidence suggests that trade in intermediates is increasing. Note, however, that in the presence of inter-sectoral linkages, the non-tradability of intermediates does not imply that upstream sector producers are shielded from international competition.
where \( P_h \) (\( h = 1, 2 \)) are the prices of the two goods and \( M \) is nominal disposable income (to be defined later).

2.2. Production sector

There are three primary inputs in the economy which we call labour (\( L \)), capital (\( K \)) and land (\( Z \)), whose rates of return are respectively denoted by \( w, r \) and \( q \). The intermediate good comes in a continuum of horizontally differentiated varieties and is produced in the upstream sector by a mass of firms which is endogenously determined by free-entry and exit in response to profit opportunities. The intermediate varieties are produced with a fixed requirement in terms of capital (\( \gamma \)) and a variable per-unit labour input requirement (\( \delta \)). The resulting decreasing average cost technology generates an incentive for specialisation leading to a one-to-one correspondence between the mass of firms and that of available varieties. Thus, the labour requirement of a typical firm \( i \) is \( l_i = \delta x_i \) and its profit will be given by

\[
\pi_i = p_i x_i - w_i \delta x_i - r \gamma,
\]

where \( x_i \) and \( p_i \) are the firm's output and price and \( w_i \) is the wage it pays its workforce.

The intermediate varieties are used in the production of the final goods by the downstream industries and are assembled into a composite input according to a CES technology. The price index for the differentiated varieties is

\[
P_x = \left( \int_{x \in N} p_i^{1-\sigma} di \right)^{1/1-\sigma},
\]

where \( N \) is the mass of firms in the industry and \( \sigma > 1 \) is the elasticity of substitution between varieties, which implies that no single variety is an essential input. A well known implication of this technology is that the productivity of the intermediate input is increasing in the range of available varieties. Hence, entry of new firms, by introducing new varieties of specialised inputs, will generate increasing returns at the aggregate level, thus enhancing the production efficiency of the final good sectors.

The downstream industries produce the two homogenous consumer goods \( Y_1 \) and \( Y_2 \).
which we call ‘high’ and ‘low-tech’ respectively and which are freely traded in world markets. Labour is not directly required as a primary factor in the production of either good. Instead, both commodities are produced using a basket of the intermediate varieties and the two other primary factors. For a given set of intermediate inputs, both final goods are produced with a constant returns to scale Cobb-Douglas technology

\[ Y_h = (\alpha_h^{-\alpha_h} \beta_h^{-\beta_h} \lambda_h^{-\lambda_h}) K_h^\alpha_h Z_h^\beta_h X_h^{1-h} \quad (h=1,2) \]

where \( 0 < \alpha_h < 1, \ 0 < \beta_h < 1 \) and \( \lambda_h = 1 - \alpha_h - \beta_h \). We shall assume that the intermediates are used most intensively in the production of the high-tech good \( Y_1 \) (i.e. \( \lambda_1 > \lambda_2 \)) and that factor \( K \) is used most intensively in the production of the low-tech good \( Y_2 \) (i.e. \( \alpha_1 < \alpha_2 \)). Thus, \( X \) and \( K \) are the ‘extreme’ factors and \( Z \) is the ‘middle’ one. We do not at this stage impose any restriction on the relative size of \( \beta_1 \) and \( \beta_2 \). Given this production function, the minimum total cost of producing \( Y_h \), in the two final goods sectors, is

\[ C_h = Y_h \left( r^{-\alpha_h} q^{\beta_h} P_h^{\lambda_h} \right), \quad (5) \]

Since the downstream industries are perfectly competitive, they produce where price equals average cost. The small open economy assumption implies that the two final good prices \( P_h \) are determined in the world market. From (5) we obtain

\[
\begin{cases}
    P_x = \left( r^{-\alpha_1} q^{-\beta_1} P_1 \right)^{\frac{1}{\alpha_1}} ,
    \\
    P_x = \left( r^{-\alpha_2} q^{-\beta_2} P_2 \right)^{\frac{1}{\alpha_2}} ,
\end{cases} \quad (6)
\]

that give the amount the two industries are respectively willing to pay for the intermediate input in terms of their own output price and the return to the other factors. Given (6) the two industries demands for the intermediates, land and capital are respectively given by
\[
\begin{align*}
X_h^d &= \lambda_h Y_h^s \frac{P_h}{P_x}, \\
Z_h^d &= \beta_h Y_h^s \frac{P_h}{q}, \\
K_h^d &= \alpha_h Y_h^s \frac{P_h}{r}. 
\end{align*}
\] (7)

For convenience, we use the low-tech good as the numeraire and set \( P_2 = 1 \) in the following analysis.

Finally, given (4), we apply Sheppard’s Lemma to (5) to obtain a system of demand equations for the varieties of the differentiated good by the final good sectors,

\[
x_i = (X_1^d + X_2^d \left( \frac{P_1}{P_x} \right)^{-\sigma}). 
\] (8)

### 2.3. Factor markets

The markets for capital and land are perfectly competitive with resource constraints respectively given by

\[
Z = Z_1^d + Z_2^d, 
\] (9)

and

\[
K = K_1^d + K_2^d + K_x^d, 
\] (10)

where \( K_x^d = \gamma N \) is the intermediate sector’s demand for capital.

The labour market in the intermediate sector is unionised (this aspect of the model is identical to Alesina and Perotti, 1997. See also Rama, 1994). For simplicity we assume that unions have monopoly over wages with equilibrium employment being determined by firms. As in Alesina and Perotti (1997), we assume that there is a mass \( J \) of unions whose membership is symmetric. Thus, a typical union will have a mass of members \( \overline{L}_j = \overline{L} / J \),
where \( J \) is the total labour endowment of the country, and will set wages for a mass of firms \( N/J \). Clearly, given the assumed symmetry between firms, the union will set the same wage for all the firms it covers. Note that \( J \) gives an inverse measure of the degree of centralisation in the wage setting process, with smaller values of \( J \) corresponding to higher degrees of centralisation and with the typical union setting the wage for a larger mass of firms. Unionisation will result in equilibrium unemployment. Thus, given the symmetry between unions, each unions will have some unemployed members.

The objective function of a typical union \( j \) can be obtained from (1) and is given by the expected utility of its typical member

\[
V_j = \frac{L_j}{L_j} \left( I - \tau \right) w_j + \frac{\bar{L}_j - L_j}{P} b + \frac{\bar{L}_j - L_j}{P} \bar{V},
\]

where \( L_j \) is the mass of union \( j \)'s members who are employed, \( \tau \) and \( b \) are respectively the labour income tax rate and the unemployment benefit set by the government and \( P = p_1^\mu p_2^{1-\mu} \) is the consumer price index. Note that we are assuming that unemployment benefits are not taxed and \( b \) is considered as net transfer.

Given the symmetry between the firms covered by a union, a typical union \( j \) will face a labour demand of \( L_j = N J_I \). Thus, total industry demand for labour will be given by

\[
L = JL_j = N J_I.
\]

Hence, the labour market resource constraint is given by \( L \leq \bar{L} \).

2.4. Government sector

The government is a provider of welfare protection in the form of unemployment benefits, financed through factor income taxation. The government budget constraint is therefore given by

\[
b(\bar{L} - JL_j) = \pi(w, JL_j) + \rho(q\bar{K}) + \phi(q\bar{Z}),
\]
where \( \rho \) and \( \phi \) are the tax rates on capital and land incomes and \( w_j \) is the wage rate set by each union \( j \).

2.5. Foreign sector

Finally, the trade balance equation that closes the model is

\[
P_1(Y_1^s - Y_1^d) + P_2(Y_2^s - Y_2^d) = 0.
\]

which, given the definition of income \( M \) provided below, is implied by the other market equilibrium conditions and the government budget constraint described above.

3. Equilibrium

Given equations (3) and (8), the optimal price rule for a typical firm \( i \) covered by union \( j \) will be given by

\[
p_{ji} = \frac{\sigma}{\sigma - 1} \delta w_j.
\]

Clearly, since all firms have the same variable input requirement \( \delta \) and given that each firm covered by the same union faces the same wage, it follows that

\[
p_{ji} = p_j, \quad \forall i.
\]

Adopting, for simplicity, the normalisation \( \frac{\sigma - 1}{\sigma} = \delta \), firms’ optimal price setting rule can be written as

\[
p_j = w_j. \tag{15}
\]

In the free-entry equilibrium, each firm will break-even. Setting (3) equal to zero and using (15) we obtain

\[
x_j = x = \sigma \gamma \frac{r}{w_j}, \tag{16}
\]

which gives the optimal output scale of a typical firm belonging to union \( j \). Due to the use of capital as a fixed input requirement, (16) depends on relative factor prices. However, for any given \( w \) and \( r \), the optimal output scale is constant and equal for all firms. Hence, the size of
the market affects neither the mark-up over marginal cost nor the scale at which individual varieties are produced, with the extent to which each firm exploits internal increasing returns to scale depending only on the elasticity of substitution between varieties\textsuperscript{7}.

It will prove useful to note that given (15) and (16), the CES quantity index dual to (4) implies the industry zero profit condition,

$$X = \gamma \sigma \frac{r}{p_x} N.$$  \hfill (17)

Equilibrium wages are determined by the monopoly unions. Two scenarios can be envisaged. In the first, unions are assumed to be small enough (i.e. $J$ is large enough) for each union not to internalise the government budget constraint. In the second scenario, unions are large enough to internalise the link between taxation paid by their members and the unemployment benefits they receive.

When unions are not strong enough to internalise the negative effects of their actions, a typical union $j$ will maximise (11) to find the optimal wage

$$w_j = \frac{(b + P_\tilde{V})}{(1 - \tau)(1 - \frac{1}{\epsilon_j})},$$  \hfill (18)

where $\epsilon_j = \frac{dL_j}{dw_j} \frac{w_j}{L_j}$ is the wage elasticity of demand for labour facing union $j$. A ceteris paribus increase in $\tau$, by reducing the after tax wage, induces the unions to bid up the nominal wage. An increase in $b$ (which, together with $\tilde{V}$, can be seen to represent the reservation wage) increases union members’ alternative utility and thus induces the unions to increase their wage demands. The union’s mark-up on the reservation wage is inversely related to $\epsilon_j$, that is as the elasticity of its labour demand with respect to wage falls, the union’s monopoly power increases. It is straightforward to show that (see Appendix A1)

\textsuperscript{7} This result stems from the constant elasticity of substitution assumption and from the lack of strategic interaction between firms. In this model therefore all scale effects at the firm level will work through changes in the product range, $N$. 

11
\[ \varepsilon_j = \sigma - \frac{\sigma - 1}{J}. \]  

(19)

Since \( \frac{d\varepsilon_j}{dJ} > 0 \), as \( J \) increases and unions become smaller, their mark-up falls and they reap smaller rents.

When the institutional bargaining framework is very centralised, as in ‘corporatist’ economies, unions’ internalisation of the government budget constraint will lead to lower wage demands. In this case, unions will maximise the following objective function, which is obtained by combining (11) and the government budget constraint in (13),

\[ \begin{aligned} 
V_j &= \frac{1}{pL_j} \left\{ w_jL_j + \rho r \frac{K}{J} + \phi q \frac{Z}{J} + (L_j - L_j) \right\}, 
\end{aligned} \]  

(11')

to obtain the optimal wage

\[ w_j = \frac{p\bar{V}}{(1 - \frac{1}{\bar{\varepsilon}_j})}, \]  

(18')

which is independent of the income tax and of the unemployment benefit rates\(^8\) and is unambiguously lower than that in (18).

Clearly, the fact that both unions and firms are symmetric implies that, under both wage setting regimes, \( w_j = w \ \forall j \), that is all employed workers will receive the same wage in equilibrium. As a result, the government budget constraint in (13) can be re-written as

\[ b(\bar{L} - L) = \tau(wL) + \rho(rK) + \phi(qZ). \]

Finally, given (16), (12) can be re-written as

\[ L = \gamma(\sigma - l) \frac{r}{w} N, \]  

(20)

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\(^8\) This result would have been the same even if we had assumed the limiting case of \( J=1 \).
which, together with (17) and the normalisation \( \delta = \frac{\sigma - 1}{\sigma} \), implies \( wL = \delta P_x X \).

### 3.1. Circular causation, intermediate wages, welfare and trade pattern

Denoting a relative change by a hat (\(^\hat{\cdot}\)) over a variable, we proceed by writing the structural equations of the model developed above in terms of proportional changes.

Totally differentiating (6) and solving for changes in \( q \) and \( r \) we obtain

\[
\hat{q} = -\theta_q \hat{P}_x, \\
\hat{r} = \theta_r \hat{P}_x,
\]

where \( \theta_q = \frac{\alpha_2 \lambda_1 - \alpha_1 \lambda_2}{\alpha_2 \beta_1 - \alpha_1 \beta_2} \) and \( \theta_r = \frac{\beta_2 \lambda_1 - \beta_1 \lambda_2}{\alpha_2 \beta_1 - \alpha_1 \beta_2} \). From (4) and given (15), we obtain

\[
\hat{P}_x = \frac{1}{1 - \sigma} \hat{N} + \hat{w},
\]

and the zero profit condition in (17) yields

\[
\hat{P}_x + \hat{X} = \hat{N} + \hat{r}.
\]

Manipulating the factor demand equations in (7) and the resource constraints in (9) and (10) we obtain

\[
\hat{r} = a\hat{q} + (1 - a)(\hat{P}_x + \hat{X}),
\]

where \( a = \left(\frac{\alpha_2 \lambda_1 - \alpha_1 \lambda_2}{\beta_2 \lambda_1 - \beta_1 \lambda_2}\right) \left(\frac{qZ}{rK}\right) \) (see Appendix A3).

Equations (21)-(24) can be solved to eliminate \( \hat{r}, \hat{q}, \hat{P}_x \) and \( \hat{X} \) to obtain the following relationship between relative changes in the intermediate sector wage and mass of firms:

---

\(^9\) See Appendix A2 for the existence and uniqueness of a solution in levels as well as for derivation of (21).
\[
\hat{N} = \left( \frac{1}{\sigma - 1} + \frac{1-a}{a(\theta_r + \theta_q)} \right)^{-1} \hat{w}.
\]  

(25)

It is tedious but straightforward to show that a sufficient condition for the expression in brackets on the (25) to be positive is

**Condition 1:**

(1.a) \[ \frac{\alpha_1}{\alpha_2} < \frac{\beta_1}{\beta_2} < 1 < \frac{\lambda_1}{\lambda_2}, \text{ and} \]

(1.b) \[ \sigma < 1 + \frac{\beta_2 - \beta_1}{\alpha_2 \beta_1 - \alpha_1 \beta_2}. \]

In addition to what was originally assumed, Condition (1.a) implies that the ‘middle’ factor \( Z \) is used more intensively in the low-tech sector. This sector is therefore relatively intensive in both primary factors. Condition (1.b) requires the elasticity of substitution between the intermediate varieties to be “not too large”\(^{10}\). Given the positive relationship between the price index in (4) and \( \sigma \), this is equivalent to requiring sufficiently strong aggregate economies of scale. It is useful to note that (1.a) implies that \( \theta_q > 0 \), \( \theta_r > 1 \) and \( a > 0 \), and (1.b) is equivalent to \( \sigma < \theta \), and also implies \( a < 1 \). Hence, under these conditions \( 0 < a < 1 \). Thus, when these two restrictions hold, an increase in the intermediate sector’s wage results in a wider range of intermediate varieties being produced\(^{11}\). Before proceeding it is worth to clarify further the role of these restrictions. Even a casual observation of real world industries suggests that condition (1.a) is highly plausible. It may indeed be fairly common for high-tech sectors to be intensive in highly specialised knowledge-based inputs but to use relatively less intensively other factors such as physical capital and land. Hence, the restriction in (1.a) could be

\(^{10}\) Note that this is not too restrictive and \( \sigma > 2 \) can be obtained as long as \( (\beta_2 - \beta_1)(\alpha_2 \beta_1 - \alpha_1 \beta_2) > 1 \) which is satisfied if \( (\alpha_1 / \alpha_2) > (\beta_2 / \beta_1) > (1+\alpha_2)/(1+\alpha_1) \).

\(^{11}\) Clearly we could have assumed that \( X \) and \( Z \) are the ‘extreme’ factors and \( K \) is the ‘middle’ factor. In this case, Condition 1 would be 1.a) \( (\beta_2 / \beta_1) < (\alpha_2 / \alpha_1) < 1 < (\lambda_1 / \lambda_2) \), and 1.b) \( \sigma < \sigma^* \) where \( \sigma^* \) is derived by imposing the positivity condition on the right-hand-side of (25). Also note, that an equivalent condition can be obtained in the case (available on request from the authors) in which the fixed cost in the monopolistically competitive sector is in terms of labour. In both the cases discussed in this footnote, the qualitative conclusions of this section would not be altered.
imposed from the start as a plausible assumption, in which case Condition (1.b) alone would be sufficient for \( \frac{dN}{dw} > 0 \).

In order to be able to analyse the effects of policy changes and labour market shocks on the general equilibrium of the model, we shall need to determine the effect of wage changes on the country’s income and on the trade balance.

Total income is given by

\[
M = (1 - \tau)wL + (1 - \rho)rK + (1 - \phi)qZ + b(L - L)
\]

and provides a measure of the country’s welfare. Using the government budget constraint

\[
b(L - L) = \tau(wL) + \rho(rK) + \phi(qZ)
\]

and recalling that \( wL = \delta P_x X \), it can be re-written as

\[
M = \delta P_x X + rK + qZ,
\]

which implies

\[
\hat{M} = \left( \frac{\delta P_x X}{M} \right) \hat{P_x} + \hat{X} + \left( \frac{rK}{M} \right) \hat{r} + \left( \frac{qZ}{M} \right) \hat{q}.
\]  

(26)

Substituting from (21)-(24), it is straightforward to show that Condition 1 is sufficient for

\[
\frac{dM}{dN} > 0
\]

(see Appendix A4). Hence, given that

\[
\frac{dM}{dN} = \frac{dM}{dw} \frac{dw}{dN},
\]

it follows that Conditions 1 is sufficient to ensure that higher wage rates result in an increase in the country’s income.

The excess supply of the high-tech good is

\[
Y_i^e = Y_i^d - Y_i^d
\]

which, upon substitution for the right-hand-side components, yields

\[
Y_i^e = \left( \frac{\beta_2}{\beta_2 \lambda_1 - \beta_1 \lambda_2} - \mu \delta \right) \left( \frac{P_x X}{P_i} \right) - \left( \frac{\lambda_2}{\beta_2 \lambda_1 - \beta_1 \lambda_2} + \mu \right) \left( \frac{qZ}{P_i} \right) - \mu \left( \frac{rK}{P_i} \right).
\]  

(27)

Equation (27) can be totally differentiated to derive (see Appendix A5)

\[
dY_i^e = \left[ \left( \frac{\beta_2}{\beta_2 \lambda_1 - \beta_1 \lambda_2} - \mu \delta \right) \left( \frac{P_x X}{P_i} \right) \left( \frac{\theta_r + a \theta_q}{1 - a} \right) \right] \\
+ \left( \frac{\lambda_2}{\beta_2 \lambda_1 - \beta_1 \lambda_2} - \mu \left( \frac{1 - a}{a} \right) \left( \frac{qZ}{P_i} \right) \theta_q \right) \left( \frac{1 - a}{a(\theta_r + \theta_q)} \right) \hat{N}
\]

(28)

Hence,
Condition 2: \[ \frac{dY_i^e}{dN} > 0 \text{ if Condition 1 holds and } 0 < \mu < \frac{a\lambda_3}{(1-a)(\beta_3\lambda_1 - \beta_2\lambda_2)}. \]

Given that \[ \frac{dY_i^e}{dw} = \frac{dY_i^e}{dN} \frac{dN}{dw}, \] it follows that Conditions 1 and 2 are sufficient to ensure that increases in the wage rate result in a pattern of specialisation which is more biased towards the high-tech good.

Thus, these results imply that there are circumstances in which an increase in the wage paid to the intermediate sector’s workers will lead to a rise in: (i) the range of intermediate varieties \( \frac{dN}{dw} > 0 \); (ii) the country’s welfare \( \frac{dM}{dw} > 0 \); and (iii) the degree of specialisation in the high-tech sector \( \frac{dY_i^e}{dw} > 0 \).

To highlight the intuition behind these results, it is useful to start by holding the mass of firms in the intermediate sector constant. For any given \( N \), an increase in wage will have two immediate effects on the economy. First, it will ceteris paribus increase income and the demand for both final goods, with no effect on the relative demand for factors. Second, the higher wage will result in an increase in each firm’s price and in the industry price index. With the other factor prices and all factor endowments held constant, this will change the relative demand for factors and result in adjustments in final goods outputs. In the first instance, as the intermediate good becomes more expensive, there will be a substitution in both final good sectors towards capital and land and an overall shift of resources towards the low-tech good will follow. This, however, will not be the end of the adjustment process since, when the number of factors exceeds the number of commodities, shocks to factor markets will generate changes in the output mix that will clear only as many factor markets as there are outputs (see Jones and Easton, 1983). Thus, if outputs adjust to clear markets for the extreme factors, excess supply or demand for the middle factor will result; it is straightforward to show that under Condition 1.a., the market for factor \( Z \) will be in an excess demand situation. This will lead to subsequent adjustments to the other factors’ demands. At constant \( N \), the demand for the differentiated input will increase, resulting in the entry of new firms in the upstream intermediate sector. This will, in turn, activate two main forces.

First, the increase in \( N \) will put pressure on the relative cost of capital, since the entry of new firms will decrease the capital stock available for final goods production, i.e.
As a result, the relative cost of the good which uses capital less intensively (that is the high-tech good) will fall, thus increasing the relative demand for and supply of this good. Second, the wider range of available varieties will result in a higher productivity of the intermediate good which will benefit the production of both final goods. However, the cost of production of the high-tech good will decline relatively more since this good uses the intermediate input more intensively. Thus, these two forces will lead to an increase in production and export of the high-tech relative to the low-tech good and induce a process of circular causation of rising demand for intermediates and entry of new firms into the upstream sector\(^\text{12}\).

An interesting special case of this model is the ‘specific-factor’ model\(^\text{13}\), where one factor is mobile between sectors and each of the other factors is used specifically by one of the sectors. In our case, the ‘middle’ factor \(Z\) would be the mobile factor, while \(X\) and \(K\) would only be used in sector one and two respectively (i.e. \(\alpha_1 = 0\) and \(\lambda_2 = 0\)). A sufficient condition for \(\frac{dN}{dw} > 0\) would then be (a) \(\frac{\beta_1}{\beta_2} < 1\), and (b) \(\sigma < 1 + \frac{\beta_2 - \beta_1}{\beta_1 (1 - \beta_2)}\). Thus, even in this case, circumstances will exist where an increase in the intermediate sector wage will lead to a virtuous circle of entry and specialisation in the high-tech sector.

4. **UNIONS’ POWER, WELFARE STATE POLICIES AND SPECIALISATION**

In this section we shall analyse the effects of different policies on the country’s pattern of international specialisation and level of income. Our main focus in this paper is on how welfare state redistributive policies affect a country’s economic performance in the presence of different labour market institutional arrangements. To this end, we shall consider (i) a policy shock, in the form of changes in the rate of unemployment benefits; and (ii) a shock to the institutional setting of the labour market, in the form of a change in the degree of centralisation of the wage setting process (i.e. changes in the mass – and hence monopoly power – of unions). Both of these shocks will have implications for the government budget constraint.

---

\(^{12}\) The assumption that the fixed cost in the intermediate sector is in terms of capital does strengthen this process of circular causation but is not crucial to it. Hence, the conclusions of this section would not be qualitatively altered by assuming that the fixed cost was in terms of labour, instead.

\(^{13}\) See for instance Jones (1971) and Neary (1978).
The issue therefore arises as to what will their overall effects be when different tax instruments are used to offset them.

As discussed in the previous section, the degree of centralisation of the wage setting process plays an important role in determining the nature of the policy effects. We shall therefore distinguish between the cases in which the unions internalise and do not internalise the effects of their actions on taxation.

4.1. Unions do not internalise the government budget constraint

When unions are small enough for each not to internalise the link between taxation and unemployment benefit, the wage rate set by each union \( j \) is given by equation (18).

Exploiting the symmetry between unions, the optimal wage rule in equation (18) in the non-internalisation case (denoted by superscript \( NI \)) can be written as

\[
\Omega^{NI} = \left(1 - \frac{1}{\varepsilon}\right) (1 - \tau) w - b = P \bar{V},
\]

which upon total differentiation yields (see Appendix A6)

\[
\Omega_w^{NI} dw + \Omega_{\tau}^{NI} d\tau + \Omega_{J}^{NI} dJ + \Omega_b^{NI} db = 0. \tag{29}
\]

Totally differentiating the government budget constraint in (13) and rearranging terms yields (see Appendix A7)

\[
G_w'^{NI} dw + G_{\tau}'^{NI} d\tau + G_{\rho}'^{NI} d\rho + G_{\phi}'^{NI} d\phi + G_{b}'^{NI} db = 0. \tag{30}
\]

The government can offset the budgetary consequences of an exogenous change in the mass of unions, while maintaining a given level of benefit rate, by changing any of the tax instruments. If Condition 1 holds, (29) and (30) imply\(^{14}\)

---

\(^{14}\) Equations (31) and (32) below give \( dw/dJ \) and \( dw/db \) as solutions obtained from equations (29) and (30) when \( db \) and \( dJ \) are set to zero respectively and only the relevant tax instrument is allowed to adjust. See Appendix A7 for determining the signs of the derivatives in (31) and (32).
Clearly, regardless of the way the shock is offset, an increase in the number of unions will reduce the equilibrium wage.

It is also interesting to analyse the effects on the equilibrium wage of changes in unemployment benefit, financed through the different tax instruments. From equations (29) and (30), when Condition 1 holds, we obtain (see Appendix A7)

\[
\begin{align*}
\frac{dw}{dJ}_{\tau = 0, \phi = 0} &= \frac{dJ}{dJ}_{\tau = 0, \phi = 0} = -\frac{\Omega_{j}^{NI}}{\Omega_{w}^{NI}} < 0, \\
\frac{dw}{dJ}_{\tau = 0, \phi = 0} &= \frac{G_{\tau}^{*} \Omega_{j}^{NI}}{\Omega_{w}^{NI}} < 0
\end{align*}
\]  

Thus

\[
\begin{align*}
\frac{dw}{db}_{\tau = 0, \phi = 0} &= \frac{db}{db}_{\tau = 0, \phi = 0} = -\frac{\Omega_{j}^{NI}}{\Omega_{w}^{NI}} > 0, \\
\frac{dw}{db}_{\tau = 0, \phi = 0} &= \frac{G_{\tau}^{*} \Omega_{j}^{NI}}{\Omega_{w}^{NI}} > 0
\end{align*}
\]  

The tax instrument used does not affect the nature of the effect of a change in the unemployment benefit rate on the equilibrium wage. Thus

**Proposition 1:** Regardless of the tax instrument used to offset the effects of the shock on the government budget constraint, Condition 1 is sufficient for both (i) a reduction in the mass of unions (i.e. an increase in unions’ monopoly power), and (ii) a rise in unemployment benefit to result in an increase in the upstream sector’s general equilibrium wage.
Using the above results we can examine how these shocks affect the country’s disposable income and pattern of export. Given (26) and (28), (31) and (32) imply the following

\[
\begin{align*}
\frac{dM}{dJ} &= \frac{dM}{dN} \frac{dN}{dw} \frac{dw}{dJ} < 0,
\end{align*}
\]

and

\[
\begin{align*}
\frac{dY^C_i}{dJ} &= \frac{dY^C_i}{dN} \frac{dN}{dw} \frac{dw}{dJ} < 0,
\end{align*}
\]

Thus:

**Proposition 2:** Regardless of the tax instrument used to offset the effects of a shock on the government budget constraint, Condition 1 is sufficient for both (i) a reduction in the mass of unions (i.e. an increase in unions’ monopoly power), and (ii) a rise in unemployment benefit to result in an increase in the country’s income and in the extent to which it specialises in the high-tech good.

Finally, it is interesting to analyse the effects on total labour demand of changes in the institutional arrangements in the labour market and of the welfare provision. From equations (12) we obtain

\[
\frac{dL}{dw} = l_i \frac{dN}{dw} + N \frac{dl_i}{dw},
\]
It can be shown that $\frac{dL}{dw} > 0$ for all $\sigma \geq 2$ (see Appendix A8). Since $\frac{dl_i}{dw} < 0$, this clearly implies that the effect of an increase in wage on entry dominates in most cases its negative effect on firms’ labour demand. Thus:

**Proposition 3:** When Condition 1 holds, those factors (e.g. an increase in unemployment benefits or a rise in the monopoly power of unions) which have a positive impact on the equilibrium wage in the upstream industry are also likely to lead to an overall decrease in unemployment.

The results outlined in Propositions 1-3 cast doubts on the general validity of the conventional wisdom outlined in the introduction. Depending on the nature of final goods technology, in the presence of vertical linkages between sectors an increase in unemployment benefit rate and/or an increase in union power may – by increasing the intermediate sector wage – lead to an increase in disposable income and may intensify the specialisation of production and trade in the high-tech sector\(^{15}\). In the next subsection we shall examine the case of a centralised wage setting process.

### 4.2. Unions internalise the government budget constraint

When unions are large enough to internalise the link between taxation and unemployment benefit (denoted by superscript $I$), the wage rate set by each union $j$ is given by equation (18), which can be rewritten as $\Omega^I = \left(1 - \frac{1}{\epsilon}\right)w = P\bar{V}$. Upon total differentiation, this yields (see Appendix A9)

$$\Omega^I_w dw + \Omega^I_J dJ = 0.$$  \hspace{1cm} (36)

As already mentioned, in this case changes in taxation and unemployment benefits do not affect the wage and labour cost. From (36), we obtain

\(^{15}\) Under the conditions discussed in Section 3.1, these results also hold in the specific-factor case.
\[
\frac{dw}{dJ} = - \frac{\Omega_j'}{\Omega_w'} < 0. \tag{37}
\]

**Proposition 4:** When unions internalise the government budget constraint: (1) changes in the degree of social protection do not have any effect on the equilibrium wage, regardless of the tax instrument used to offset them; and (2) Condition 1 is sufficient for a reduction in the mass of unions (i.e. a rise in unions’ monopoly power) to result in an increase in the upstream sector’s general equilibrium wage.

Given (26) and (28), (37) implies that the effects of a change in \( J \) on \( M \) and \( Y_i^r \) are qualitatively identical to that in (33), whereas a change in \( b \) does not have any effect on the former variables since the unions’ wage is independent of the unemployment benefit, hence

\[
\begin{align*}
\frac{dM}{db} = 0, \\
\frac{dY_i^r}{db} = 0.
\end{align*} \tag{38}
\]

Thus:

**Proposition 5:** With highly centralised unions, regardless of the tax instrument used to offset the effects of the shock on the government budget constraint, (1) Condition 1 is sufficient for a reduction in the mass of unions (i.e. a rise in unions’ monopoly power) to result in an increase in the country’s income and in the extent to which it specialises in the high-tech good; and (2) an increase in unemployment benefit will have no effect on income and trade pattern.

5. **SUMMARY OF THE RESULTS AND CONCLUSIONS**

In this paper we have examined the role of economy-wide increasing returns to scale in shaping the relationship between welfare state policies and economic performance in a world
with free-trade in final goods. We develop a model of a small open economy characterised by unionised labour markets and vertical linkages between two downstream final good sectors and an upstream sector producing highly specialised intermediate inputs. The main finding of the paper is that plausible circumstances exist in which proactive welfare state policies have a positive impact on the depth of the division of labour, on national welfare and on the extent of specialisation on high-tech sectors which intensively utilise highly specialised intermediate inputs.

Through their monopoly power, unions will shift the burden of income taxation on to the wage and labour costs. In recent years a widespread consensus has emerged in the literature about the fact that – in so doing – unions may be most disruptive at intermediate levels of centralisation, that is when they are strong enough to reap substantial rents but not large enough to take account of the links between their actions and the government budget constraint. Our analysis is consistent with the hump-shaped relationship between the degree of distortion of fiscal policy and the levels of centralisation of labour markets highlighted by Alesina and Perotti (1997): higher degrees of centralisation lead to higher wages – up to when unions are large enough to internalise the government budget constraint, at which point increases in taxation will lead to smaller increases in wages than at low levels of centralisation. Hence, in the presence of unionisation, larger welfare states – by implying higher taxation – would normally lead to higher distortions. However, our results point to the possibility that the ‘disruption’ of higher wages may not always have negative implications for the overall performance of an economy. This is because the input-output structure of the economy could be such that higher wages in the intermediate sector may lead to a higher demand for intermediates, thus generating a virtuous circle of entry, higher welfare and greater specialisation in high-tech sectors.

These results have two major implications. First, strong but not very centralised unions, with a high degree of shifting of taxation, may have a positive impact on the country’s level of wages, income, and specialisation in high-tech sectors. It follows that in the presence of openness, corporatist countries with highly centralised labour markets may exploit less the potential economy-wide increasing returns stemming form vertical linkages in production and may therefore be less specialised in high-tech goods. Second, the benefits of the welfare state

16 See for instance Calmfors and Driffill (1988), Summers, Gruber and Vergara (1993), and Rama (1994).
are higher if it is financed through labour income taxes that, because of their stronger impact on wages, will enhance the strength of the virtuous circle.

Caution should be used when drawing policy conclusions in the presence of complementarities whose nature is highly sensitive to changes in environment. Our aim here was not to obtain generally valid results about the interaction between welfare state policies and economic performance for open economies. Instead, we wanted to stress that when more complex economic structures are allowed for – such as those characterised by inter-sectoral linkages – the ‘obviousness’ of commonly accepted conclusions may have to be critically scrutinised.

Many different extensions present themselves. We have not analysed the role of trade policy and have assumed an exogenous absence of trade barriers. Trade policy analysis could be carried out to highlight the implications of trade liberalisation. Our model already suggests that a large welfare state may be compatible with openness and foster specialisation in high-tech goods. The introduction of an explicit trade policy analysis may help to shed further light on the empirical regularity (e.g. Rodrik, 1998) of a positive correlation between government expenditure (of which welfare spending is a high proportion, at least in industrial economies) and openness to foreign trade. It is also interesting to examine the implications of exposing factor markets to global competition.
References


APPENDIX

A1. Derivation of (18), \( \varepsilon_j = \sigma - \frac{\sigma - 1}{J} \).

To derive \( \varepsilon_j \), we need to define union \( j \)'s labour demand. This is given by \( L_j = \int_{i \in N_j} l_i \, di \) which in the symmetric equilibrium can be written as \( L_j = \frac{N}{J} l_{ji} \), where \( l_{ji} \) denotes employment of firm \( i \) in union \( j \). Using \( l_{ji} = \delta x_{ji} \), recalling that (7) and (8) imply \( x_{ji} = (\lambda_j P_j Y_i + \lambda_2 P_2 Y_2) P_x^{-\sigma} p_{ji}^{-\sigma} \), and partitioning the CES price index in (4) as

\[
P_x = \left( \int_{i=0}^{N_j} p_i^{-\sigma} \, di + \int_{i=N_j}^{N} p_i^{-\sigma} \, di \right)^{-1},
\]

we obtain

\[
\frac{dL_j}{dw_j} = \frac{N}{J} \delta \frac{x_j}{w_j} \left( (\sigma - 1) \frac{dP_x}{dw_j} \frac{w_j}{P_x} - \sigma \right).
\]

Using

\[
\frac{dP_x}{dw_j} = \frac{N}{J} \left( \frac{p_j}{P_x} \right)^{-\sigma} \frac{dp_j}{dw_j}, \quad p_j = p_j = w_j \text{ and } \left( \frac{p_j}{P_x} \right)^{1-\sigma} = N,
\]

yields (18).

A2. Existence and uniqueness of a solution in levels, and derivation of (21).

The model, for a given symmetric long-run equilibrium in the intermediate input industry, consists of the following equations

\[
P_x = (r^{-\alpha_1} q^{-\beta_1} P_j)^{\frac{1}{\gamma}}, \quad \text{(A2.1)}
\]

\[
P_x = (r^{-\alpha_2} q^{-\beta_2} P_2)^{\frac{1}{\gamma}}; \quad P_2 = 1, \quad \text{(A2.2)}
\]

\[
P_x = N^{\frac{1}{1-\sigma}} w, \quad \text{(A2.3)}
\]

\[
P_x X = (\gamma \sigma) rN, \quad \text{(A2.4)}
\]

\[
rK = \left( \frac{\alpha_2 \lambda_2 - \alpha_1 \lambda_1}{\beta_2 \lambda_1 - \beta_1 \lambda_2} \right) qZ + \left( \frac{1}{\sigma} - \frac{\alpha_2 \beta_1 - \alpha_1 \beta_2}{\beta_2 \lambda_1 - \beta_1 \lambda_2} \right) P_x X, \quad \text{(A2.5)}
\]

\[
L = \gamma (\sigma - 1) r w \quad \text{N}, \quad \text{(A2.6)}
\]

\[
b(L - L) = \tau(wL) + \rho(rK) + \phi(qZ), \quad \text{(A2.7)}
\]

\[
\varepsilon = \sigma - \frac{\sigma - 1}{J}, \quad \text{(A2.8)}
\]
and either
\[ w = \frac{(b + P\tilde{V})}{(1 - \tau)(1 - \frac{1}{\epsilon})}; \quad P = \frac{P_1 P_2^{1-\mu}}{}, \] (A2.9) 

or
\[ w = \frac{P\tilde{V}}{(1 - \frac{1}{\epsilon})}; \quad P = \frac{P_1 P_2^{1-\mu}}{}, \] (A2.9')

where: (A2.1) and (A2.2) are from equation (6) with the normalisation \( P_2 = I \); (A2.3) is obtained from (4) and (15) in the symmetric equilibrium; (A2.4) is the zero profit condition in the intermediate input industry in (17); (A2.5) is the overall resource constraint (A3.4) obtained in A3 below; (A2.6) is the labour requirement equation in (20); (A2.7) is the government budget constraint; (A2.8) is the labour demand elasticity in (19) which was derived in A1 above; and (A2.9) and (A2.9') are the union wage setting equations in (18) and (18') with \( P \) defined as the Cobb-Douglas price index.

For given values of the exogenous variable \((b, J, P_1, P_2, \tilde{V})\) and the parameters \((\alpha_1, \alpha_2, \beta_1, \beta_2, \lambda_1, \lambda_2, \gamma, \sigma)\), (A2.1)-(A2.9) can, in principle, be solved to determine the values of the endogenous variables \((P_x, r, q, N, X, w, L, \epsilon)\), and one of the tax rates \((\tau, \rho, \phi)\). To examine the existence and uniqueness of the solution we note that for any given \(w\) equations (A2.1)-(A2.5) can be solved for \((P_x, r, q, N, X)\) and it is this part of the model that, due to its nonlinearities in \(N\), could generate problems of existence and uniqueness. We therefore focus on reducing these equations to one which determines \(N\) and then using that equation to derive a sufficient condition for a unique solution to exist. To do so, first solve (A2.1) and (A2.2) to obtain

\[ r = \left(\frac{P^{\beta_1}}{P^{\beta_2}}\right)^{\frac{\gamma}{\beta_1}} P^{\theta_r}, \] (A2.10)

\[ q = \left(\frac{P^{\alpha_1}}{P^{\alpha_2}}\right)^{\frac{\gamma}{\alpha_1}} P^{\theta_q}, \] (A2.11)

where \(\theta_q = \frac{\alpha_2 \lambda_1 - \alpha_1 \lambda_2}{\alpha_2 \beta_1 - \alpha_1 \beta_2}\), \(\theta_r = \frac{\beta_2 \lambda_1 - \beta_1 \lambda_2}{\alpha_2 \beta_1 - \alpha_1 \beta_2}\), and \(\Delta = -(\alpha_2 \beta_1 - \alpha_1 \beta_2)\). Note that (A2.10) and (A2.11) are used to derive (21). Substituting for \(P_x\) from (A2.3) in these equations implies

28
\[ q/r = cN^{\theta_r + \theta_q}, \quad \text{(A2.12)} \]

where \( c = w^{-\theta_r - \theta_q}\left(P_1^\gamma (\alpha + \beta_1) P_2^\gamma (\alpha + \beta_1)^2\right) > 0 \). Next, substituting from (A2.4) into (A2.5) and rearranging using the above expressions for \( \theta_r \) and \( \theta_q \), we obtain

\[
\left( \frac{Z}{K} \right) \left( \begin{array}{c} \theta_q \\ \theta_r \end{array} \right) r + \left( \frac{\gamma \sigma}{K} \right) \left( \frac{1}{\sigma} - \frac{1}{\theta_r} \right) \left( \begin{array}{c} \gamma \\ \gamma + 1 \end{array} \right) N = I, \quad \text{(A2.13)}
\]

which, together with (A2.12), implies

\[
\left( c \frac{Z}{K} \right) \left( \begin{array}{c} \theta_q \\ \theta_r \end{array} \right) r^{\theta_q + \theta_r} \left( \frac{\gamma \sigma}{K} \right) \left( \frac{1}{\sigma - \theta_r} \right) N = I. \quad \text{(A2.14)}
\]

Although the left-hand-side of (A2.14) is nonlinear in \( N \), Condition 1 specified in the text – following equation (25) – is sufficient for a unique \( N \) to solve the equality in (A2.14). This is because under that condition \( \theta_r > 0 \), \( \theta_q > 0 \), and \( \theta_r + \theta_q > \sigma - 1 \) which imply the left-hand-side of (A2.14) to be zero at \( N=0 \), to tend to infinity as \( N \to \infty \), and to be positive and monotonically increasing in \( N \).

**A3. Derivation of (24),** \( \hat{r} = a\hat{q} + (1-a)\left(\hat{P}_c + \hat{X}\right) \).

Using (7), (9), (10) and noting that \( X^i = X^d_i + X^d_2 \) and \( K^d = \gamma N \), we obtain the following three resource constraints:

\[
\begin{align*}
X^i &= X^d_i + X^d_2 = \lambda_1 P_i Y_i + \lambda_2 P_i^2 Y_i^2 \\
Z &= Z^d_i + Z^d_2 = \beta_1 q Y_i + \beta_2 \frac{1}{r} Y_i^2, \\
\bar{K} &= K^d_i + K^d_2 + K^d_x = \alpha_1 P_i Y_i + \alpha_2 \frac{1}{r} Y_i^2 + \gamma N.
\end{align*}
\quad \text{(A3.1)}
\]
Solving the first two equations for $Y_i^*$ and $Y_2^*$ yields,

$$p_i Y_i^* = \frac{\beta_j P_r X - \lambda_j q Z}{\beta_j \lambda_i - \beta_i \lambda_j}, \quad (A3.2)$$

and

$$p_2 Y_2^* = \frac{\lambda_j q Z - \beta_i P_r X}{\beta_i \lambda_i - \beta_i \lambda_2}. \quad (A3.3)$$

Substituting these into the third equation, and noting from (17) that $\gamma N = \frac{1}{\sigma} \frac{P_r}{r} X^*$, yields the ‘overall’ resource constraint,

$$rK = \left( \frac{\alpha_i \lambda_i - \alpha_j \lambda_j}{\beta_i \lambda_i - \beta_j \lambda_j} \right) q Z + \left( \frac{1 - \alpha_i \beta_j - \alpha_j \beta_i}{\beta_j \lambda_i - \beta_i \lambda_j} \right) p_i X^* . \quad (A3.4)$$

which imposes a restriction on the values of the three factors and, regardless of the structure of factor intensities, should hold for all $Y_i^*$ and $Y_2^*$. Equation (24) is obtained by totally differentiating (A3.4) where $a = \left( \frac{\alpha_i \lambda_i - \alpha_j \lambda_j}{\beta_i \lambda_i - \beta_j \lambda_j} \right) \frac{q Z}{rK}$. Note that, given (A3.4), it follows that

$$1 - a = \left( \frac{1 - \alpha_i \beta_j - \alpha_j \beta_i}{\beta_j \lambda_i - \beta_i \lambda_j} \right) \frac{p_i X^*}{rK} .$$

It is useful to note that a comparison between $a$, $\theta_r$ and $\theta_q$ implies $\frac{a \theta_r}{\theta_q} = \frac{q Z}{rK}$ and $1 - a = \left( \frac{1}{\theta_r} \right) \left( \frac{p_i X^*}{rK} \right)$ where the latter shows that $a < 1$ iff $\sigma < \theta_r$.

A4. Derivation of $\frac{dM}{dN} > 0$.

First, substituting from (21) and (24) in (26) we can re-write the latter as,

$$\dot{M} = \left( \frac{\delta P_r X}{M} \right) \left( \frac{\theta_r + a \theta_q}{1 - a} \right) + \left( \frac{rK}{M} \right) \theta_r - \left( \frac{q Z}{M} \right) \theta_q$$

which, using $\frac{a \theta_r}{\theta_q} = \frac{q Z}{rK}$ derived in A3 above, can be written as
\[ \dot{M} = \left( \frac{\partial P X}{M} \left( \frac{\theta_r + a \theta_q}{1-a} \right) + \left( 1-a \right) \frac{rK}{M} \right) \dot{P}_x. \]

Next, substituting from (21) and (23) into (24) we obtain \( \dot{P}_x = \left( \frac{1-a}{a(\theta_r + \theta_q)} \right) \dot{N} \). As a result, by ensuring that \( \theta_q > 0, \theta_r > 0 \) and \( 0 < a < 1 \), Condition 1 is also sufficient for \( \frac{dM}{dN} > 0 \).

A5. Derivation of \( \frac{dY'^*_i}{dN} > 0 \).

Using the values of supply and demand for good 1 given by equations (A3.2) and (2), we obtain

\[ P_i Y'^*_i = P_i Y'^*_i - P_i Y'^*_d = \frac{\beta \beta_i P_i X - \lambda_i qZ}{\beta_i \beta_i - \beta_i \beta_i} - \mu \left( \frac{\partial P X + rK + qZ}{P_1} \right) \] which can be rearranged as equation (27):

\[ Y'^*_i = \left( \frac{\beta_2}{\beta_2 \beta_1 - \beta_1 \beta_2} - \mu \delta \left( \frac{P_1 X}{P_1} \right) \right) \left( \frac{\lambda_2}{\beta_2 \beta_1 - \beta_1 \beta_2} + \mu \left( \frac{qZ}{P_1} \right) \right) \frac{rK}{P_1}. \]

Totally differentiating this gives

\[ dY'^*_i = \left( \frac{\beta_2}{\beta_2 \beta_1 - \beta_1 \beta_2} - \mu \delta \left( \frac{P_1 X}{P_1} \right) \right) \left( \frac{\theta_r + a \theta_q}{1-a} \right) \frac{\dot{P}_x}{\dot{X}} - \left( \frac{\lambda_2}{\beta_2 \beta_1 - \beta_1 \beta_2} + \mu \left( \frac{qZ}{P_1} \right) \right) \frac{\dot{q}}{\dot{P}_x} \right) \frac{rK}{P_1}. \]

which, using equations (21) and (24) and noting that \( \frac{a \theta_r}{\theta_q} = \frac{qZ}{rK} \), can be rearranged as

\[ dY'^*_i = \left( \frac{\beta_2}{\beta_2 \beta_1 - \beta_1 \beta_2} - \mu \delta \left( \frac{P_1 X}{P_1} \right) \right) \left( \frac{\theta_r + a \theta_q}{1-a} \right) \frac{\dot{P}_x}{\dot{X}} + \left( \frac{\lambda_2}{\beta_2 \beta_1 - \beta_1 \beta_2} - \mu \left( \frac{1-a}{a} \right) \right) \frac{qZ}{P_1} \theta_q \frac{\dot{P}_x}{\dot{X}}. \]

Given that \( \dot{P}_x = \left( \frac{1-a}{a(\theta_r + \theta_q)} \right) \dot{N} \), we obtain equation (28):

\[ dY'^*_i = \left( \frac{\beta_2}{\beta_2 \beta_1 - \beta_1 \beta_2} - \mu \delta \left( \frac{P_1 X}{P_1} \right) \right) \left( \frac{\theta_r + a \theta_q}{1-a} \right) + \left( \frac{\lambda_2}{\beta_2 \beta_1 - \beta_1 \beta_2} - \mu \left( \frac{1-a}{a} \right) \right) \frac{qZ}{P_1} \theta_q \left( \frac{1-a}{a(\theta_r + \theta_q)} \right) \dot{N}. \]

31
Thus, the sufficient conditions for $\frac{dY^e}{dN} > 0$, in addition to Condition 1, are

$$\frac{\beta_2}{\beta_2 \lambda_1 - \beta_1 \lambda_2} - \mu \delta > 0 \quad \text{and} \quad \frac{\lambda_2}{\beta_2 \lambda_1 - \beta_1 \lambda_2} - \mu \left( \frac{1-a}{a} \right) > 0.$$ The former is always satisfied since

$$\frac{\beta_2}{\beta_2 \lambda_1 - \beta_1 \lambda_2} > 1 > \mu \delta > 0$$ holds and the latter simply requires the propensity to consume good 1 from income to be sufficiently small, i.e. $0 < \mu < \frac{a \lambda_2}{(1-a)(\beta_2 \lambda_1 - \beta_1 \lambda_2)}$, hence Condition 2.

**A6. Derivation of (29).**

Totally differentiating $\Omega^{NI}$ noting that $\epsilon = \sigma - \frac{1 - J}{J}$ yields

$$\Omega_w^{NI} dw + \Omega_t^{NI} d\tau + \Omega_j^{NI} dJ + \Omega_b^{NI} db = 0,$$

where

$$\Omega_w^{NI} = \left( 1 - \frac{1}{\epsilon} \right) (1 - \tau) > 0; \quad \Omega_t^{NI} = -(1 - \tau)w < 0; \quad \Omega_j^{NI} = \frac{(\sigma - 1)(1 - \tau)w}{(J - (\sigma - 1)\gamma)^2} > 0; \quad \Omega_b^{NI} = -1.$$

**A7. Derivation of (30), (31) and (32).**

Using (20), the government budget constraint can be re-written as

$$\gamma(\sigma - 1) \left( \frac{b}{w} \right) rN + \rho(rK) + \phi(qZ) - b \bar{L} = 0,$$

whose total differentiation yields:

$$G_a^e dN + G_q^e dq + G_r^e dr + G_w^e dw + G_t^e d\tau + G_p^e d\rho + G_\phi^e d\phi + G_b^e db = 0$$

where

$$G_a^e = \gamma(\sigma - 1)(\tau + b / w)r; \quad G_q^e = \gamma(\sigma - 1)(\tau + b / w)N + \rho K; \quad G_w^e = \phi Z; \quad G_t^e = -\gamma(\sigma - 1)brN / w^2; \quad G_r^e = \gamma(\sigma - 1) rN; \quad G_p^e = \bar{K}; \quad G_\phi^e = q \bar{Z}; \quad G_b^e = \gamma(\sigma - 1) rN / w - \bar{L}.$$
We define $G_{w}^{\ast}dw = G_{n}^{\ast}dN + G_{q}^{\ast}dq + G_{r}^{\ast}dr + G_{w}^{\ast}dw = \left( G_{n}^{\ast} + G_{q}^{\ast} \frac{dq}{dN} + G_{r}^{\ast} \frac{dR}{dN} \right) \frac{dN}{dw} + G_{w}^{\ast} \right) dw$

where $G_{w}^{\ast}$ can be evaluated as follows. From (21), and given that $\hat{P}_x = \left( \frac{1-a}{a(\theta, + \theta_q)} \right) \hat{N}$, we obtain $\hat{r} = \left( \frac{(1-a)\theta}{a(\theta, + \theta_q)} \right) \hat{N} \quad \text{and} \quad \hat{q} = \left( \frac{(1-a)\theta}{a(\theta, + \theta_q)} \right) \hat{N}$ which imply $\frac{dr}{dN} = \left( \frac{(1-a)\theta}{a(\theta, + \theta_q)} \right)$, and

$\frac{dq}{dN} = \left( \frac{(1-a)\theta}{a(\theta, + \theta_q)} \right)$. Equation (25) can be written as $\frac{dN}{dw} = \frac{N}{W} \left( 1 + \frac{1-a}{a(\theta, + \theta_q)} \right)$. These, together with the expressions for the partial derivatives $G_{n}^{\ast}$, $G_{q}^{\ast}$, $G_{r}^{\ast}$ and $G_{w}^{\ast}$ can be substituted in the above expression for $G_{w}^{\ast}$ to yield:

$$G_{w}^{\ast} = \begin{pmatrix} \gamma(\sigma-1)(\tau+b/w) \left( 1 - \frac{a\theta}{a(\theta, + \theta_q)} \right) \frac{\theta}{N} \\
+ \left( \gamma(\sigma-1)(\tau+b/w) \right) N + \rho \bar{K} \left( \frac{(1-a)\theta}{a(\theta, + \theta_q)} \right) \frac{\theta}{N} \right) \left( \frac{1}{W} \left( 1 + \frac{1-a}{a(\theta, + \theta_q)} \right) \right) \\
- \frac{\gamma(\sigma-1)bN}{w^2} + \left( \gamma(\sigma-1) \right) \frac{N}{W} \left( 1 + \frac{1-a}{a(\theta, + \theta_q)} \right) \\
\end{pmatrix}$$

To determine the signs of derivatives in (31), note that the only ambiguous expression is

$$G_{w}^{\ast} - G_{r}^{\ast} \frac{\Omega_{w}^{NI}}{\Omega_{r}^{NI}} = \begin{pmatrix} \gamma(\sigma-1)(\tau+b/w) \left( 1 - \frac{a\theta}{a(\theta, + \theta_q)} \right) \frac{\theta}{N} \\
+ \left( \gamma(\sigma-1)(\tau+b/w) \theta, rN + \rho \theta, r \bar{K} - \phi \theta, q \bar{Z} \left( \frac{(1-a)}{a(\theta, + \theta_q)} \right) \right) \left( \frac{1}{W} \left( 1 + \frac{1-a}{a(\theta, + \theta_q)} \right) \right) \\
- \frac{\gamma(\sigma-1)bN}{w^2} + \left( \gamma(\sigma-1) \right) \frac{N}{W} \left( 1 + \frac{1-a}{a(\theta, + \theta_q)} \right) \\
\end{pmatrix}$$

Starting from an initial situation in which Condition 1 holds and $\rho = \phi$, it can be seen that all terms in $\{ \}$ on the right-hand-side are positive since $\rho \theta, r \bar{K} - \phi \theta, q \bar{Z} = \rho(\theta, r \bar{K} - a \theta, r \bar{K}) > 0$, given that $\frac{a\theta}{\theta_q} = \frac{q\bar{Z}}{r\bar{K}}$. Also, given equation (18), $b = \left( \frac{1}{\epsilon} \right) \left( 1 - \tau \right) w - P \bar{V}$, which can be
used to write the last two terms as $\left(\frac{\gamma(\sigma-1)rN}{w^2}\right)\left(\left(1 - \frac{1}{\varepsilon}\right)w - \left(1 - \frac{1}{\varepsilon}\right)1 - \tau \right)w + P\tilde{V} > 0$.

A8. Derivation of (35), $\frac{dL}{dw} = l_i \frac{dN}{dw} + N \frac{dl_i}{dw} > 0$.

From (20), $L = \gamma(\sigma-1)rN/w$ whose total differentiation implies $\dot{L} = \dot{r} + N - \dot{w}$. Using (21) and noting that $\dot{P}_i = \left(\frac{1-a}{a(\theta_r + \theta_q)}\right) \dot{N}$, this can be rewritten as $\dot{L} = \left(\frac{(1-a)\theta_r}{a(\theta_r + \theta_q)}\right) + 1 \dot{N} - \dot{w}$.

Substituting for $\dot{N}$ from (25) implies $\dot{L} = \left[\left(\frac{(1-a)\theta_r}{a(\theta_r + \theta_q)}\right) + 1 \left(\frac{1}{\sigma-1} + \frac{1-a}{a(\theta_r + \theta_q)}\right)\right]^{-1} \dot{w}$, or

or

or

the right-hand-side of which is positive as long as $\sigma > 2$ and Condition 1 holds so that $0 < a < 1$ and $\theta_r > 1$.

A9. Derivation of (36) and (37).

Totally differentiating $\Omega^J$ and noting that $\varepsilon = \sigma - \frac{\sigma - 1}{J}$ yields $\Omega^N_w \, dw + \Omega^N_J \, dJ = 0$, where

$\Omega^N_w = \left(1 - \frac{1}{\varepsilon}\right) > 0; \quad \Omega^N_J = \frac{(\sigma - 1)w}{(J \sigma - (\sigma - 1))^2} > 0$. 
